

The Weak Gravity Conjecture and the Viscosity Bound with Six-Derivative Corrections

Aaron J. Amsel* and Dan Gorbonos†
Theoretical Physics Institute
University of Alberta
Edmonton, Alberta, Canada T6G 2G7

The weak gravity conjecture and the shear viscosity to entropy density bound place constraints on low energy effective field theories that may help to distinguish which theories can be UV completed. Recently, there have been suggestions of a possible correlation between the two constraints. In some interesting cases, the behavior was precisely such that the conjectures were mutually exclusive. Motivated by these works, we study the mass to charge and shear viscosity to entropy density ratios for charged AdS_5 black branes, which are holographically dual to four-dimensional CFTs at finite temperature. We study a family of four-derivative and six-derivative perturbative corrections to these backgrounds. We identify the region in parameter space where the two constraints are satisfied and in particular find that the inclusion of the next-to-leading perturbative correction introduces wider possibilities for the satisfaction of both constraints.

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*Electronic address: aamsel@ualberta.ca

†Electronic address: dan.gorbonos@mail.huji.ac.il

I. INTRODUCTION

The weak gravity conjecture (WGC) [1] and the shear viscosity to entropy density bound (KSS) [2] have been suggested as tests to distinguish theories in the landscape from theories that belong to the swampland [3–5]. As argued in [3], any quantum gravity theory imposes certain constraints on low energy physics, so that not every effective theory can be UV completed. Thus, effective theories can be classified as “good” theories that admit a valid UV completion (landscape) or “bad” theories that cannot be consistently completed (swampland).

Even though both conjectures (WGC and KSS) serve to place constraints on effective theories, there are some important differences between the two. The WGC was formulated for gravity in asymptotically flat spacetime, while the KSS bound was formulated for asymptotically AdS spacetime. The WGC deals with physical, global quantities that are usually defined at the asymptotic boundary of the spacetime (mass and charge), while the KSS bound deals with local quantities which are defined at the horizon (viscosity and entropy density). Nevertheless, there have been attempts to extend the applicability of the WGC to asymptotically AdS backgrounds as well [5–9]. As explained further below, these works also made the intriguing suggestion that these conjectures might in fact be correlated to each other in some way. In this work, we shall further explore the interplay of the two constraints in a toy model of an effective theory with four-derivative and six-derivative corrections.

One implication of the WGC is that higher derivative corrections in a consistent theory of quantum gravity should reduce the mass to charge ratio of extremal black holes¹ [1, 10]. We give here a brief version of the argument for this statement. For classical charged black holes (e.g. Reissner-Nordström), the minimal value of the mass to charge ratio is achieved when the black hole is extremal, while going below this minimal value creates a naked singularity. We assume that the existence of a large number of stable black hole states is unnatural unless there is a global symmetry (e.g. supersymmetry) that protects them from quantum corrections and/or decay. One may then further argue that such a symmetry should not exist in a consistent theory of quantum gravity [1]. Therefore, quantum corrections should take the parameters of the black hole away from their classical extremal values. As an additional consequence, any black hole (even at extremality) should be allowed to decay. Let us choose units in which the classical value of the mass to charge ratio at extremality is 1. Suppose that quantum corrected black holes satisfy $M/Q > 1$ and that they are unstable. Then a given black hole will decay into two black holes with masses M_1, M_2 and charges Q_1, Q_2 such that $M_1 + M_2 < M$, $Q_1 + Q_2 = Q$. According to our assumption, the decay products should also satisfy $M_i/Q_i > 1$ ($i = 1, 2$) (see Figure 1, which reproduces a similar figure from [10]). Since $M_i < M$, the quantum correction is larger, which implies $M_i/Q_i > M/Q$. Then it follows that $M_1 + M_2 > M$, which contradicts our assumption about the instability of this branch. Hence, the second branch (where $M/Q < 1$) is unstable and is expected to occur in a consistent quantum gravity theory.

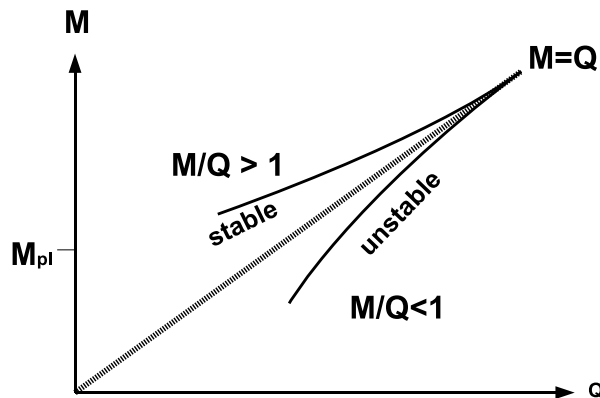


FIG. 1: The two a-priori possibilities for the quantum corrections to extremal black holes. The unstable branch is the one which is expected to occur in consistent theories of quantum gravity.

¹ In what follows, the term “WGC” will be meant only to refer to this particular aspect of the conjecture.

The perturbed heterotic string states were given in [1] as evidence in favor of the conjecture. These states are non-supersymmetric and approach the line $M = Q$ from below as expected from the WGC. An analysis of the mass to charge ratio for general four-derivative corrections to non-supersymmetric black holes in asymptotically flat spacetime was carried out in [10]. The results were consistent with the conjecture for those cases in which the values of the four-derivative couplings are known (four dimensions). Further support for the conjecture was given in [11], based on semi-classical considerations.

If the WGC is true, it is expected to apply also to non-extremal black holes close to extremality, since we expect theories to behave smoothly as the parameters vary. The conjecture was examined for four-dimensional non-extremal black holes with two electric charges, which are solutions that correspond to fundamental strings with generic momentum and winding on an internal circle [12]. The results showed that for this case the mass to charge ratio is smaller compared to the uncorrected ratio for any value of the charges where a regular black hole solution exists. These results were later extended to d dimensions [13] with the same conclusions.

The KSS bound was presented as a conjecture for field theories that have a holographic dual in [2]. The conjecture suggests that the ratio of the coefficient of shear viscosity η to entropy density s has a lower bound and that this lower bound is $1/4\pi$. The bound is saturated for boundary field theories in the limit of infinite 't Hooft coupling λ and number of colors N_c . Such theories are dual to Einstein gravity (without corrections). The authors of [2] also gave a general argument that the ratio should be greater than some constant of order one. A short version of the argument is as follows. The product of the energy ϵ of a particle in the fluid and its mean free time τ_{mft} is, according to Heisenberg's uncertainty principle, $\epsilon\tau_{mft} > \hbar$. The viscosity is proportional to $n\epsilon\tau_{mft}$, where n is the density of particles. The entropy density is also proportional to n , with $s \sim k_B n$. Then the viscosity to entropy density ratio is $\eta/s \sim \hbar/k_B$, and since we take $\hbar = k_B = 1$, the constant is of order one.

Another piece of evidence supporting the conjecture arose from the explicit computation of the leading α' correction for type IIB string theory compactified to five dimensions (R^4 corrections) [14–16]. This computation showed that the correction increased the shear viscosity to entropy density ratio.

However, more general computations in higher derivative gravity showed that the KSS bound can be violated, although the crucial sign of the coefficient in front of the higher derivative correction is in general undetermined. Examples of models where there is a possible violation of the KSS bound were given in [4, 6, 8, 17–27]. Violations of the bound might be related to inconsistencies of the boundary theory, for example by introduction of ghosts [28]. Even if violation of the KSS bound occurs in some models, there may still exist an improved lower bound. A bound of $4/25\pi$ was proposed in [29] based on causality violation in the CFT. Note that this bound is based on the consistency of the boundary theory and is still quite close to the original bound. For the purposes of this work, we will usually take the bound to be $1/4\pi$. Even if the bound were different, as explained above, it would be of the same order and the qualitative conclusions often remain the same. We will, however, point out below some instances where the behavior does change if the bound is relaxed to $4/25\pi$.

Suggestions of correlation between the two constraints appeared in [5–9]. In [5], examples with four-derivative corrections were studied in a five-dimensional asymptotically AdS spacetime. The examples suggested that the bounds cannot be satisfied simultaneously. In particular, the WGC and KSS bound require opposite signs of the coefficients of certain corrections in four-derivative gravity. The precise statement depends of course on the relations (if any) between the coefficients. In certain cases of interest, Weyl anomaly matching shows that the coefficients are proportional to the difference of the two central charges a, c in the dual four dimensional CFT. As a result, it appeared that the WGC required $c - a > 0$, but this is exactly the condition that implies violation of the KSS bound. This is true for naturally appearing corrections like the Gauss-Bonnet term

$$R_{GB}^2 \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \quad (1.1)$$

or Weyl-tensor-squared corrections [5] (motivated by the general form of supersymmetric higher derivative actions)

$$W^2 \equiv \frac{1}{6} R^2 - \frac{4}{3} R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \quad (1.2)$$

There are some special cases in four-derivative gravity in which the two bounds are satisfied together, such as when one takes the only correction to be $R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$ [9]. However, we will not consider this term in our toy model family (as discussed below).

It is important to note that in the two examples listed above (and indeed for generic four-derivative corrections excluding the $R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$ term), the correction to the shear viscosity to entropy density ratio vanishes in the extremal limit. In such cases, it is natural to consider η/s for non-extremal cases in order to see the direction from which the bound is approached. To gain additional perspective on the apparent tension between the bounds, it is also interesting though to find further examples in which the shear viscosity to entropy density ratio does not vanish at extremality. As we will see, the first order at which this phenomenon occurs is in many cases at the level of curvature-cubed corrections (see also [30]).

Six-derivative gravity is also important for studying the effects of higher derivative corrections that capture more parameters of the dual CFT at finite 't Hooft coupling and number of colors [31, 32]. For example, curvature-cubed corrections in the bulk are required to fully characterize the energy flux one-point function of the CFT [33]. Furthermore, since curvature-cubed terms break supersymmetry, these theories are relevant to the study of non-supersymmetric CFTs [32].

In general, when we want to consider the WGC and KSS bound for black holes with parameters closer to the Planck scale, the next correction after four-derivatives will be six-derivatives. Of course, all the higher derivative corrections are important for the full quantum gravity description in the vicinity of the Planck scale. We can, however, consider an intermediate small region just beyond the regime where the four-derivative terms dominate. In this region, the six-derivative corrections become of the same order as the four-derivative corrections, but higher orders are still negligible. Below, we shall study the two constraints in this region, which is a further step towards the quantum gravity regime.

In order to explore new possibilities for satisfying both constraints in six-derivative actions, we take a toy model that includes two curvature-cubed terms which are invariant under field redefinitions:

$$I_1 = R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\lambda\rho} R^{\lambda\rho}{}_{\mu\nu}, \quad I_2 = R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\lambda\mu} R^{\sigma}{}_{\tau}{}^{\lambda}{}_{\nu}. \quad (1.3)$$

These are the only terms required to describe curvature-cubed gravity corrections (up to field redefinitions) when charge is not included [34]. For simplicity, we do not include higher derivative corrections involving the Maxwell field in our toy model. In other words, we add the charge only to the Einstein-Hilbert part of the action. We leave the general six-derivative action for future study, as there are many additional terms that involve Maxwell fields. As we will see, the two six-derivative terms that we do include already provide a much richer picture than the restriction to only four-derivative terms.

Explicitly, we take for our toy model an action of the form

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left[R - \frac{1}{4} F^2 - 2\Lambda + b_1 R^2 + b_2 R_{GB}^2 + b_3 R_{\mu\nu} R^{\mu\nu} + c_1 I_1 + c_2 I_2 \right]. \quad (1.4)$$

One may view an action of this form as an effective theory arising in a string theory α' expansion, with $b_i \sim O(\alpha')$ and $c_i \sim O(\alpha'^2)$. In theories without charge, we can eliminate the b_1 and b_3 terms by field redefinitions and write the action only with b_2 (the Gauss-Bonnet term). When we include charge, however, these terms cannot be eliminated as the field redefinitions would then generate $R F^2$ and $R_{\mu\nu} F^{\mu\lambda} F^{\nu}{}_{\lambda}$ terms [5].

The paper is organized as follows. In section II, we compute the perturbative corrections to the mass to charge ratio by applying the covariant ADT method [35, 36] to a d dimensional extension of (1.4). In section III, we compute the perturbative corrections to the shear viscosity to entropy density ratio in our toy model. Next, in section IV, we analyze the WGC and KSS bound to determine the influence of the $O(\alpha'^2)$ corrections on various cases of the action (1.4). In particular, for various toy models we provide precise plots of regions in the parameter space where the two bounds are satisfied, i.e. where the theories are “good.” Our main conclusion is that six-derivative terms give us more possibilities to satisfy the constraints and that the apparent tension observed in four-derivative gravity may disappear when viewed from the higher order perspective. We conclude with a brief discussion of our results and future directions in section V.

II. THE MASS TO CHARGE RATIO

In this section, we calculate the mass to charge ratio of a static, asymptotically AdS black hole solution of a six-derivative gravity theory specified below. We begin by giving a brief introduction to a general method (referred to as “Abbot-Deser-Tekin” or “ADT” method) for calculating the energy in higher derivative gravity [36, 37]. We then apply this procedure to the particular case of an action with terms up to curvature-cubed. Finally, we couple the theory to a gauge field and obtain a perturbative expression for the mass to charge ratio of a charged black hole solution.

A. The ADT Method

The celebrated result of ADM [38] for energy in Einstein-Hilbert gravity with asymptotically flat boundary conditions was generalized to spacetimes with a cosmological constant in [35]. These so-called “AD charges” were written in a manifestly covariant way and once again could be expressed as pure surface integrals. The method used to construct the AD charges was then further generalized to arbitrary higher curvature theories in [36, 37].

This ADT method is similar in spirit to the Landau-Lifshitz pseudotensor method for calculating energy [39] in asymptotically flat curved spacetime. In particular, one proceeds by linearizing the equations of motion with respect to a background spacetime (AdS in our case). This leads to an effective stress-energy tensor that consists of matter sources and terms higher order in the perturbation. This tensor turns out to be covariantly conserved and can thus be used to construct a conserved charge associated with an isometry of the background.

As we will see, the ADT method involves relatively little formalism and is computationally straightforward. In addition, this method has the advantage of not involving any explicit regularization or subtraction of infinities, as required in counter-term methods (see e.g. [40, 41]). Unlike Euclidean path integral techniques (e.g. [42]), the ADT framework naturally gives the gravitational mass as an integral at asymptotic infinity, without any need to identify a horizon in the interior. For perturbations that vanish sufficiently fast at asymptotic infinity², the ADT charges are exactly the same as the charges derived using the covariant phase space methods of [44–46], which in turn differ from the charges of Wald et al. [47–49] by a surface term proportional to the Killing equations.

Let us consider some arbitrary gravitational theory with equations of motion of the form

$$\Phi_{\mu\nu}(g, R, \nabla R, R^2, \dots) = \kappa \tau_{\mu\nu}, \quad (2.1)$$

where κ is the gravitational coupling and $\tau_{\mu\nu}$ is the matter stress-energy tensor. The symmetric tensor $\Phi_{\mu\nu}$, which is the analogue of the Einstein tensor, may depend on the metric, the curvature, derivatives of the curvature, and various combinations thereof. Assuming that the action is invariant under diffeomorphisms, we obtain the geometric identity $\nabla^\mu \Phi_{\mu\nu} = 0$ (the generalized Bianchi identity) and the covariant conservation of the stress tensor $\nabla^\mu \tau_{\mu\nu} = 0$.

Now, we further assume that there exists a background solution $\bar{g}_{\mu\nu}$ to the equations (2.1) with $\tau_{\mu\nu} = 0$. Then we decompose the metric as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad (2.2)$$

where we note that the deviation $h_{\mu\nu}$ is not necessarily infinitesimal, but again is required to fall off sufficiently fast at infinity. By expanding the left-hand side of (2.1) in $h_{\mu\nu}$, the equations of motion may be expressed as

$$\phi_{\mu\nu}^{(1)} = \kappa \tau_{\mu\nu} - \phi_{\mu\nu}^{(2)} - \phi_{\mu\nu}^{(3)} \dots \equiv \kappa T_{\mu\nu}, \quad (2.3)$$

where $\phi_{\mu\nu}^{(i)}$ denotes all terms in the expansion of $\Phi_{\mu\nu}$ involving i powers of $h_{\mu\nu}$ and we have defined the effective stress-tensor $T_{\mu\nu}$. It then follows from the Bianchi identity of the full theory that $\bar{\nabla}^\mu \phi_{\mu\nu}^{(1)} = 0 = \bar{\nabla}^\mu T_{\mu\nu}$.

Suppose that the background spacetime admits a timelike Killing vector $\bar{\xi}^\mu$ and let Σ be a constant-time hypersurface with unit normal n^μ . Then we can construct a conserved energy in the standard way

$$E = \int_{\Sigma} d^{d-1}x \sqrt{\bar{g}_{\Sigma}} n_{\mu} T^{\mu\nu} \bar{\xi}_{\nu}, \quad (2.4)$$

where \bar{g}_{Σ} denotes the determinant of the induced metric on Σ . Because $\bar{\nabla}^\mu (T_{\mu\nu} \bar{\xi}^\nu) = 0$, it follows that $T_{\mu\nu} \bar{\xi}^\nu = \bar{\nabla}^\nu \mathcal{F}_{\nu\mu}$ for some antisymmetric tensor $\mathcal{F}_{\nu\mu}$. The bulk integral (2.4) can therefore be rewritten as a surface integral over the boundary $\partial\Sigma$

$$E = \int_{\partial\Sigma} d^{d-2}x \sqrt{\bar{g}_{\partial\Sigma}} n_{\mu} r_{\nu} \mathcal{F}^{\nu\mu}, \quad (2.5)$$

where r_{μ} is the unit normal to the boundary.

In summary, to apply the ADT method, one linearizes the equations of motion to obtain the stress-energy tensor, and then expresses the conserved current $T^{\mu\nu} \bar{\xi}_{\nu}$ as a total derivative to find the “potential” $\mathcal{F}^{\nu\mu}$. Note that by construction, the background spacetime $\bar{g}_{\mu\nu}$ has $E = 0$.

² Here we mean that the perturbation about a solution falls off fast enough at infinity that the theory is asymptotically linear, i.e. that the linearized equations of motion are obeyed near infinity. In this case, the charges of the linearized theory can be used to obtain the charges of the non-linear theory. This condition indeed holds for the case of standard asymptotically AdS boundary conditions [43] that we consider in this work.

B. Energy in Six-Derivative Gravity

We now wish to apply the ADT procedure to a six-derivative theory that we describe below. The case of a generic four-derivative theory has been worked out in detail previously [37], so it remains to apply the method only to the curvature-cubed terms we wish to add. The only potentially non-trivial step is to rewrite the conserved current as a total derivative, but we will see that there is a simplification below.

Let us consider a theory of the form

$$S = \frac{1}{16\pi G} \int d^d x \sqrt{-g} \left[R - 2\Lambda + b_1 R^2 + b_2 (R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2) + b_3 R_{\mu\nu}^2 + c_1 R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\lambda\rho} R^{\lambda\rho}{}_{\mu\nu} + c_2 R^{\mu\nu}{}_{\rho\sigma} R^{\rho\tau}{}_{\lambda\mu} R^{\sigma}{}_{\tau}{}^{\lambda}{}_{\nu} \right] + S_m, \quad (2.6)$$

where the matter action

$$S_m = \frac{1}{16\pi G} \int d^d x \sqrt{-g} L_m \quad (2.7)$$

is at this point arbitrary. The corresponding equation of motion is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} + \Phi_{\mu\nu}^{(4)} + \Phi_{\mu\nu}^{(6)} = -\frac{16\pi G}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} = 16\pi G \tau_{\mu\nu} \quad (2.8)$$

where

$$\begin{aligned} \Phi_{\mu\nu}^{(4)} &= 2b_1 R \left(R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} \right) + (2b_1 + b_3) (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) R + b_3 \square \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) \\ &\quad + 2b_2 \left(R R_{\mu\nu} - 2R_{\mu\sigma\nu\rho} R^{\sigma\rho} + R_{\mu\sigma\rho\lambda} R_{\nu}{}^{\sigma\rho\lambda} - 2R_{\mu\sigma} R^{\sigma}{}_{\nu} - \frac{1}{4} (R_{\lambda\kappa\rho\sigma}^2 - 4R_{\rho\sigma}^2 + R^2) g_{\mu\nu} \right) \\ &\quad + 2b_3 \left(R_{\mu\sigma\nu\rho} - \frac{1}{4} R_{\sigma\rho} g_{\mu\nu} \right) R^{\sigma\rho} \end{aligned} \quad (2.9)$$

$$\begin{aligned} \Phi_{\mu\nu}^{(6)} &= \frac{1}{2} \left[-6c_1 \nabla_\kappa \nabla_\rho (R_{\mu}{}^{\rho\lambda\tau} R_{\lambda\tau}{}^{\kappa}{}_{\nu}) + 3c_2 \nabla_\kappa \nabla_\rho (R_{\mu\tau\lambda\nu} R^{\rho\tau\lambda\kappa} - R_{\mu}{}^{\tau\lambda\kappa} R^{\rho}{}_{\tau\lambda\nu}) \right. \\ &\quad + c_1 (3R_{\sigma\mu\lambda\rho} R^{\lambda\rho\alpha\beta} R^{\sigma}{}_{\nu\alpha\beta} - \frac{1}{2} R^{\sigma\gamma}{}_{\lambda\rho} R^{\lambda\rho\alpha\beta} R_{\alpha\beta\sigma\gamma} g_{\mu\nu}) \\ &\quad \left. + c_2 (3R^{\beta}{}_{\mu\rho\sigma} R^{\rho\tau}{}_{\lambda\beta} R^{\sigma}{}_{\tau}{}^{\lambda}{}_{\nu} - \frac{1}{2} R^{\beta\gamma}{}_{\rho\sigma} R^{\rho\tau}{}_{\lambda\beta} R^{\sigma}{}_{\tau}{}^{\lambda}{}_{\gamma} g_{\mu\nu}) \right] + \frac{1}{2} [\mu \leftrightarrow \nu]. \end{aligned} \quad (2.10)$$

We now look for an exact AdS solution of these equations with no matter fields. Using that in this case the Riemann tensor takes the maximally symmetric form

$$\bar{R}_{\mu\rho\nu\sigma} = \frac{2\Lambda_{eff}}{(d-1)(d-2)} (\bar{g}_{\mu\nu} \bar{g}_{\rho\sigma} - \bar{g}_{\mu\sigma} \bar{g}_{\rho\nu}), \quad (2.11)$$

we find that there can exist an AdS solution with an “effective” cosmological constant Λ_{eff} satisfying the cubic equation

$$\begin{aligned} 0 &= \frac{8}{(d-2)^3(d-1)^2} \left[2(6-d)c_1 + \frac{(6-d)(d-2)}{2} c_2 \right] \Lambda_{eff}^3 \\ &\quad - 2 \left[\frac{(d-4)(db_1 + b_3)}{(d-2)^2} + \frac{(d-3)(d-4)b_2}{(d-2)(d-1)} \right] \Lambda_{eff}^2 - \Lambda_{eff} + \Lambda. \end{aligned} \quad (2.12)$$

The perturbative solution of this equation takes the form $\Lambda_{eff} = \Lambda + \dots$, where the explicit expressions for the corrections are given in appendix A.

The next step is to linearize the equations of motion (2.8) with respect to the background AdS solution by writing $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Using the results of [37] and of appendix A, we find that the stress tensor is

$$16\pi G T_{\mu\nu} = \alpha_1 \mathcal{G}_{\mu\nu}^L + \alpha_2 \left(\bar{g}_{\mu\nu} \bar{\square} - \bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{2\Lambda_{eff}}{d-2} \bar{g}_{\mu\nu} \right) R_L + \alpha_3 \left(\bar{\square} \mathcal{G}_{\mu\nu}^L - \frac{2\Lambda_{eff}}{d-2} \bar{g}_{\mu\nu} R_L \right) \quad (2.13)$$

where

$$\mathcal{G}_{\mu\nu}^L \equiv R_{\mu\nu}^L - \frac{1}{2}\bar{g}_{\mu\nu}R_L - \frac{2\Lambda_{eff}}{d-2}h_{\mu\nu}, \quad (2.14)$$

$R_{\mu\nu}^L$ is the linearized Ricci tensor, and R_L is the linearized Ricci scalar. The coefficients are given by

$$\begin{aligned} \alpha_1 &= 1 + \frac{4d\Lambda_{eff}b_1}{d-2} + \frac{4(d-3)(d-4)\Lambda_{eff}b_2}{(d-2)(d-1)} + \frac{4\Lambda_{eff}b_3}{d-1} - \frac{48(2d-3)\Lambda_{eff}^2c_1}{(d-2)^2(d-1)^2} + \frac{36\Lambda_{eff}^2c_2}{(d-2)(d-1)^2} \\ \alpha_2 &= 2b_1 + b_3 + \frac{24\Lambda_{eff}c_1}{(d-2)(d-1)} \\ \alpha_3 &= b_3 + \frac{48\Lambda_{eff}c_1}{(d-2)(d-1)} - \frac{6\Lambda_{eff}c_2}{(d-2)(d-1)}. \end{aligned} \quad (2.15)$$

Remarkably, this result has precisely the same tensor form as the four-derivative case; the only effect of the six-derivative terms is to modify the coefficients α_i . Hence, we may simply borrow the results of [37] to obtain the potential

$$\begin{aligned} 16\pi G\mathcal{F}^{\nu\mu} &= \frac{\tilde{\alpha}_1}{2} [\bar{\xi}_\lambda \bar{\nabla}^\mu h^{\nu\lambda} - \bar{\xi}_\lambda \bar{\nabla}^\nu h^{\mu\lambda} + \bar{\xi}^\mu \bar{\nabla}^\nu h - \bar{\xi}^\nu \bar{\nabla}^\mu h + h^{\mu\lambda} \bar{\nabla}^\nu \bar{\xi}_\lambda - h^{\nu\lambda} \bar{\nabla}^\mu \bar{\xi}_\lambda + \bar{\xi}^\nu \bar{\nabla}_\lambda h^{\mu\lambda} - \bar{\xi}^\mu \bar{\nabla}_\lambda h^{\nu\lambda} + h \bar{\nabla}^\mu \bar{\xi}^\nu] \\ &+ \alpha_2 [\bar{\xi}^\mu \bar{\nabla}^\nu R_L - \bar{\xi}^\nu \bar{\nabla}^\mu R_L - R_L \bar{\nabla}^\nu \bar{\xi}^\mu] + \alpha_3 [\bar{\xi}_\lambda \bar{\nabla}^\nu \mathcal{G}_L^{\mu\lambda} - \bar{\xi}_\lambda \bar{\nabla}^\mu \mathcal{G}_L^{\nu\lambda} + \mathcal{G}_L^{\nu\lambda} \bar{\nabla}^\mu \bar{\xi}_\lambda - \mathcal{G}_L^{\mu\lambda} \bar{\nabla}^\nu \bar{\xi}_\lambda] \end{aligned} \quad (2.16)$$

with

$$\tilde{\alpha}_1 = 1 + \frac{4\Lambda_{eff}(db_1 + b_3)}{d-2} + \frac{4(d-3)(d-4)\Lambda_{eff}b_2}{(d-2)(d-1)} - \frac{48(2d-7)\Lambda_{eff}^2c_1}{(d-2)^2(d-1)^2} + \frac{12(3d-8)\Lambda_{eff}^2c_2}{(d-2)^2(d-1)^2}. \quad (2.17)$$

Note that at this stage, the expression for the energy is exact in the couplings.

Consider a static, spherically symmetric, asymptotically AdS black hole solution of the form

$$ds^2 = -f_1(r)dt^2 + \frac{dr^2}{f_2(r)} + r^2 d\Omega_{d-2,k}^2, \quad (2.18)$$

where $d\Omega_{d-2,k}^2$ is the line element of the transverse space with curvature $k = 0, 1$, and as $r \rightarrow \infty$, we have

$$f_2(r) = \frac{r^2}{\ell_{eff}^2} + k + \frac{m}{r^{d-3}} + \dots \quad (2.19)$$

Here ℓ_{eff} is the (effective) AdS radius, which is related to the cosmological constant through

$$\ell_{eff}^2 = -\frac{(d-2)(d-1)}{2\Lambda_{eff}}. \quad (2.20)$$

As noted in [37] for four-derivative gravity, it is interesting that for the class of metrics (2.18), (2.19), the α_2, α_3 terms in (2.16) fall off fast enough at infinity so that they give zero contribution to the energy. Hence, the only contribution to the energy is from the first term, which is simply the Einstein-Hilbert result with a coefficient corrected by the higher derivative terms. We now see that the same is true for our six-derivative theory (2.6). Using (2.5), (2.16) and (A3), the final expression for the energy density (to order α'^2) is

$$\begin{aligned} \mathcal{E} &= \left(1 + \frac{4d\Lambda b_1}{d-2} + \frac{4\Lambda b_3}{d-2} + \frac{4(d-4)(d-3)\Lambda b_2}{(d-2)(d-1)} - \frac{48(2d-7)\Lambda^2 c_1}{(d-2)^2(d-1)^2} + \frac{12(3d-8)\Lambda^2 c_2}{(d-2)^2(d-1)^2} \right. \\ &\quad - \frac{40d\Lambda^2 b_1^2}{9(d-2)} - \frac{8\Lambda^2 b_3^2}{9(d-2)} - \frac{4(d-4)(d-3)\Lambda^2 b_2^2}{3(d-2)(d-1)} - \frac{4(13d^2 - 73d + 120)\Lambda^2 b_1 b_2}{9(d-2)(d-1)} \\ &\quad \left. - \frac{8(d+5)\Lambda^2 b_1 b_3}{9(d-2)} - \frac{4(2d^2 - 11d + 21)\Lambda^2 b_3 b_2}{9(d-2)(d-1)} + \dots \right) \mathcal{E}_0 \end{aligned} \quad (2.21)$$

where \mathcal{E}_0 is the result for Einstein-Hilbert gravity

$$\mathcal{E}_0 = \frac{(d-2)m}{32\pi G}. \quad (2.22)$$

Note that the expression for the energy (2.21) is perturbative, since it was obtained using the perturbative solution for Λ_{eff} , (A3). This is the relevant form for our analysis since we treat the higher derivative terms as corrections to the Einstein-Hilbert action. The leading α' corrections in (2.21) match exactly the result of [5] for four-derivative gravity, which was obtained through boundary counterterm methods. While the counterterm results were only strictly valid for $d < 7$, the result (2.21) confirms the expectation that the expression for the energy in [5] holds in all d .

C. Charged AdS₅ Black Branes

To address the WGC, we want to consider a charged black brane in the theory (2.6). The simplest way to add charge is to choose the matter sector to contain just a Maxwell field with the minimal term

$$L_m = -\frac{1}{4}F^2 \quad \Rightarrow \quad \tau_{\mu\nu} = \frac{1}{32\pi G} \left(F_{\mu\lambda}F_{\nu}{}^{\lambda} - \frac{1}{4}F^2 g_{\mu\nu} \right), \quad (2.23)$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ and $F^2 = F_{\mu\nu}F^{\mu\nu}$. Hence, even though there are higher curvature terms, the matter equation of motion is still

$$\nabla^\mu F_{\mu\nu} = 0 \quad (2.24)$$

and the charge is given by the usual expression

$$\mathcal{Q} = \int_{\partial\Sigma} d^{d-2}x \sqrt{g_{\partial\Sigma}} n_\mu r_\nu F^{\mu\nu}. \quad (2.25)$$

Let us now restrict to planar ($k=0$) black branes in $d=5$ and work in units where the uncorrected AdS radius is set to $\ell=1$. We consider a general ansatz of the form

$$ds^2 = -\omega(r)dt^2 + \frac{\sigma^2(r)}{\omega(r)}dr^2 + r^2(dx^2 + dy^2 + dz^2) \quad (2.26)$$

$$A_\mu = \gamma(r)\delta_\mu^t. \quad (2.27)$$

In the absence of higher derivative corrections, the solution to the equations of motion is given by

$$\begin{aligned} \omega_{(0)}(r) &= r^2 \left(1 - \frac{r_0^2}{r^2} \right) \left(1 + \frac{r_0^2}{r^2} - \frac{q^2}{r_0^2 r^4} \right) \\ \sigma_{(0)}(r) &= 1 \\ \gamma_{(0)}(r) &= \frac{\sqrt{3}q}{r^2}. \end{aligned} \quad (2.28)$$

We assume $q^2 \leq 2r_0^6$, so that $r=r_0$ corresponds to the outer horizon. The Hawking temperature of a black brane of the type

$$ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + g_{xx}(dx^2 + \dots) \quad (2.29)$$

is given by the general formula

$$T = - \frac{\partial_r g_{tt}}{4\pi \sqrt{-g_{tt} g_{rr}}} \Big|_{r \rightarrow r_0} = \frac{\omega'(r_0)}{4\pi \sigma(r_0)}, \quad (2.30)$$

which for the above solution becomes

$$T_{(0)} = \frac{r_0}{\pi} \left(1 - \frac{q^2}{2r_0^6} \right). \quad (2.31)$$

The extremal solution is defined by the condition $T=0$, which corresponds to $q^2 = 2r_0^6$. This is also the point where the outer and inner horizons coincide. The mass is given by (2.22) with

$$m = \frac{q^2 + r_0^6}{r_0^2} \quad (2.32)$$

and using (2.25), the charge density is

$$Q = 2\sqrt{3}q. \quad (2.33)$$

Using the ansatz (2.26), (2.27) and the leading order solution, we can solve the equations of motion perturbatively in α' . The results are given in appendix B, using a scheme where the horizon radius is fixed at $r = r_0$, i.e. it is not corrected by the higher derivative contributions. This can be achieved by choosing the integration constants appropriately when solving the gravitational field equations at each order in α' . This does, however, produce corrections to the $O(r^{-2})$ “mass term” in the metric, so the m in (2.21) should properly be viewed as a function of the parameters q, r_0, b_i, c_i . The cosmological constant gets corrected as given in (A3). We also choose integration constants when solving Maxwell’s equations so that the charge is not corrected and remains as in (2.33). The Hawking temperature of the corrected solution is also given in appendix B.

To set the speed of light to be one in the dual CFT, one should actually rescale the time coordinate by a red-shift factor $t \rightarrow t/\ell_{eff}$. Equivalently, as noted in [5], we obtain the physical energy density of the field theory by simply rescaling $M \equiv \ell_{eff}\mathcal{E}$. Similarly, the temperature of the CFT is given by $T_{CFT} \equiv \ell_{eff}T$. Now, we want to compare thermodynamic quantities (like M/Q) of the uncorrected solution to those of the corrected solution³. It is important to remember that this is meaningful only if the temperature does not change when the higher derivative terms are turned on. Thus, we would actually like to write M/Q in terms of T_{CFT} instead of r_0 . To do so, we introduce a new parameter \bar{r}_0 and fix

$$T_{CFT} = \frac{\bar{r}_0}{\pi} \left(1 - \frac{q^2}{2\bar{r}_0^6} \right). \quad (2.34)$$

This relation may be solved (perturbatively) to give $r_0 = r_0(\bar{r}_0, q)$ so that we may eliminate r_0 in favor of \bar{r}_0 in all expressions. The explicit expression for \bar{r}_0 is given in appendix B. Then \bar{r}_0 is implicitly a function of (T_{CFT}, q) through (2.34) and we have ensured that the corrected and uncorrected solutions have the same temperature. The extremal case is given precisely by $q^2 = 2\bar{r}_0^6$.

Hence, the result for the mass to charge ratio is

$$\begin{aligned} \frac{M}{Q} = & \left(\frac{M}{Q} \right)_0 \left(1 + \frac{\bar{q}^6 - 186\bar{q}^4 - 60\bar{q}^2 - 200}{2(\bar{q}^2 + 1)(5\bar{q}^2 + 2)} b_1 - \frac{3(\bar{q}^4 + 5\bar{q}^2 - 2)}{(\bar{q}^2 + 1)(5\bar{q}^2 + 2)} b_2 \right. \\ & + \frac{11\bar{q}^6 - 102\bar{q}^4 - 12\bar{q}^2 - 40}{2(\bar{q}^2 + 1)(5\bar{q}^2 + 2)} b_3 - \frac{16602\bar{q}^8 - 59267\bar{q}^6 + 23548\bar{q}^4 - 7798\bar{q}^2 + 5180}{35(\bar{q}^2 + 1)(5\bar{q}^2 + 2)} c_1 \\ & - \frac{3(846\bar{q}^8 - 4521\bar{q}^6 + 6664\bar{q}^4 - 574\bar{q}^2 - 4340)}{140(\bar{q}^2 + 1)(5\bar{q}^2 + 2)} c_2 - \frac{27(\bar{q}^2 + 2)(15\bar{q}^6 + 39\bar{q}^4 - 16\bar{q}^2 - 4)}{2(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_2^2 \\ & + \frac{1860\bar{q}^{12} - 7445\bar{q}^{10} - 98484\bar{q}^8 - 154414\bar{q}^6 - 272756\bar{q}^4 + 13800\bar{q}^2 + 2000}{3(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_1^2 \\ & + \frac{4020\bar{q}^{12} + 5335\bar{q}^{10} - 143088\bar{q}^8 - 155962\bar{q}^6 - 144956\bar{q}^4 + 3864\bar{q}^2 + 560}{21(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_2^2 \\ & + \frac{1200\bar{q}^{12} - 7165\bar{q}^{10} - 24978\bar{q}^8 - 211352\bar{q}^6 - 9712\bar{q}^4 - 47568\bar{q}^2 - 800}{10(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_1 b_2 \\ & + \frac{4(2760\bar{q}^{12} + 2605\bar{q}^{10} - 156759\bar{q}^8 - 201175\bar{q}^6 - 260330\bar{q}^4 + 9660\bar{q}^2 + 1400)}{21(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_1 b_3 \\ & \left. + \frac{1200\bar{q}^{12} - 6085\bar{q}^{10} - 8202\bar{q}^8 - 84416\bar{q}^6 - 11872\bar{q}^4 - 18480\bar{q}^2 - 224}{14(\bar{q}^2 + 1)(5\bar{q}^2 + 2)^3} b_2 b_3 + \dots \right), \end{aligned} \quad (2.35)$$

where we have set $\bar{q} \equiv q/\bar{r}_0^3$ and

$$\left(\frac{M}{Q} \right)_0 = \frac{\sqrt{3}(1 + \bar{q}^2) \bar{r}_0}{64 \pi G \bar{q}}. \quad (2.36)$$

³ Note that we take the ratio of two densities, so that the volume factors cancel.

At extremality, the mass to charge ratio becomes

$$\frac{M}{Q} = \left(\frac{M}{Q}\right)_0 \left(1 - \frac{44}{3}b_1 - b_2 - \frac{16}{3}b_3 + \frac{10394}{105}c_1 + \frac{61}{70}c_2 - \frac{770}{3}b_1^2 - \frac{5}{2}b_2^2 - \frac{710}{21}b_3^2 - \frac{3764}{21}b_1b_3 - \frac{688}{15}b_1b_2 - \frac{292}{21}b_3b_2 + \dots\right) \quad (2.37)$$

where

$$\left(\frac{M}{Q}\right)_0 = \frac{3\sqrt{3}\bar{r}_0}{\sqrt{2}64\pi G}. \quad (2.38)$$

The corresponding result for black branes with spherical horizons is given in appendix C.

III. THE SHEAR VISCOSITY TO ENTROPY DENSITY RATIO

In this section, we calculate perturbative corrections to the shear viscosity to entropy density ratio of a CFT plasma dual to the static charged black brane solution (2.26,2.28). The corrections we consider are given in (1.4). We begin by giving a brief summary of the holographic method for calculating the shear viscosity of the dual CFT. We then compute the entropy density using Wald's formula for the Noether charge entropy and use these results to obtain the ratio.

A. The Shear Viscosity

To compute the viscosity, we use the prescription given in [7, 50–52]. We present here only the main steps, mostly following [7], where the four-derivative correction to the viscosity with a chemical potential was computed.

The viscosity of the boundary field theory is given by Kubo's formula:

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t, x), T_{xy}(0, 0)] \rangle. \quad (3.1)$$

The two-point function in the formula above can be expressed as a retarded Green's function of T_{xy} :

$$G_{xy,xy}^R(\omega, \mathbf{k}) = -i \int d^4x e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})} \theta(t) \langle [T_{xy}(t, x), T_{xy}(0, 0)] \rangle \quad (3.2)$$

so that

$$\eta = - \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{xy,xy}^R(\omega, 0). \quad (3.3)$$

The boundary operator T_{xy} is coupled to a metric perturbation (graviton) h^{xy} in the bulk. Hence, we can perform the Green's function computation on the metric perturbation $g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{xy}(t, r)$. It turns out that the equation of motion of h_x^y is that of a minimally coupled scalar in the case of the Einstein-Hilbert action. Higher derivative corrections enter as effective modifications of the coefficients of this scalar equation. We define

$$\phi(r, t, \mathbf{x}) \equiv h_x^y(r, t, \mathbf{x}) = \int \frac{d^4k}{(2\pi)^4} \phi_k(r) e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}} \quad (3.4)$$

and expand the action to second order in $\phi(r, t)$. This effective action is denoted by $I_\phi^{(2)}$. Next by computing the radial canonical momentum

$$\Pi(r) \equiv \frac{\delta I_\phi^{(2)}}{\delta \phi'_k} \quad (3.5)$$

we find the retarded Green's function

$$G_{xy,xy}^R = - \lim_{r \rightarrow \infty} \frac{\Pi(r)}{i\phi_k(r)}. \quad (3.6)$$

For theories without derivatives of the curvature, the effective action is of the general form

$$I_\phi^{(2)} = \frac{1}{16\pi G} \int \frac{d^4 k}{(2\pi)^4} dr \left(A(r) \phi_k'' \phi_{-k} + B(r) \phi_k' \phi_{-k}' + C(r) \phi_k' \phi_{-k} \right. \\ \left. + D(r) \phi_k \phi_{-k} + E(r) \phi_k'' \phi_{-k}'' + F(r) \phi_k'' \phi_{-k}' \right) + \text{boundary terms}. \quad (3.7)$$

Note that while one technically requires the appropriate boundary terms so that the action is well-defined, it turns out that their explicit form is not required to obtain the viscosity. The canonical conjugate momentum to ϕ is then

$$\Pi(r) = \frac{1}{8\pi G} \left[\left(B - A - \frac{1}{2} F' \right) \phi' - (E \phi'')' \right]. \quad (3.8)$$

Since in the limit $\omega \rightarrow 0$ the equation of motion is

$$\partial_r \Pi = 0, \quad (3.9)$$

we can compute the Green's function at any point. In particular, we demand infalling boundary conditions on $\phi(r)$ at the horizon (to avoid a singularity there), namely

$$\phi(r, t)|_{r=r_0} = \phi(v), \quad (3.10)$$

where v is the Eddington-Finkelstein coordinate defined as $dv = dt + \sqrt{-\frac{g_{rr}}{g_{tt}}} dr$. Then we find that [53]

$$\partial_r \phi(r, t) = \sqrt{-\frac{g_{rr}}{g_{tt}}} \partial_t \phi(r, t) \Rightarrow \partial_r \phi_k = -i\omega \sqrt{-\frac{g_{rr}}{g_{tt}}} \phi_k \quad (3.11)$$

gives a convenient formula for the viscosity [7]

$$\eta = \lim_{\omega \rightarrow 0} \frac{\Pi(r)}{i\omega \phi(r)} \Big|_{r=r_0} = \frac{1}{8\pi G} [\kappa_2(r_0) + \kappa_4(r_0)], \quad (3.12)$$

with

$$\kappa_2(r) = \sqrt{-\frac{g_{rr}}{g_{tt}}} \left(A(r) - B(r) + \frac{F'(r)}{2} \right), \quad \kappa_4(r) = \left(E(r) \left(\sqrt{-\frac{g_{rr}}{g_{tt}}} \right)' \right)'. \quad (3.13)$$

Now we are ready to apply this prescription to the action (1.4). We calculate the effective action (3.7) for the perturbation ϕ using the metric ansatz (2.26) as the background metric. The effective coefficients in the action $A(r), B(r), E(r), F(r)$ are given in appendix D. Substitution into the formula (3.12) gives

$$\eta = \frac{r_0^3}{16\pi G} \left(1 + \frac{2b_1}{\sigma(r_0)^3} \left[\left(\sigma'(r_0) - \frac{6\sigma(r_0)}{r_0} \right) \omega'(r_0) - \sigma(r_0) \omega''(r_0) \right] + \frac{b_3}{\sigma(r_0)^3} \left[\left(\sigma'(r_0) - \frac{3\sigma(r_0)}{r_0} \right) \omega'(r_0) \right. \right. \\ \left. \left. - \sigma(r_0) \omega''(r_0) \right] - \frac{2b_2 \omega'(r_0)}{r_0 \sigma(r_0)^2} - \frac{6c_1 \omega'(r_0)}{r_0^2 \sigma(r_0)^5} [(\sigma(r_0) + 4r_0 \sigma'(r_0)) \omega'(r_0) - 2r_0 \sigma(r_0) \omega''(r_0)] \right. \\ \left. - \frac{3c_2}{4r_0^2 \sigma(r_0)^6} [\omega'(r_0)^2 (4\sigma(r_0)^2 + 3r_0 \sigma(r_0) \sigma'(r_0) + 4r_0^2 \sigma'^2(r_0) - r_0^2 \sigma(r_0) \sigma''(r_0)) \right. \\ \left. + r_0 \omega'(r_0) (r_0 \sigma^2(r_0) \omega^{(3)}(r_0) - (5r_0 \sigma(r_0) \sigma'(r_0) + 3\sigma^2(r_0)) \omega''(r_0)) + r_0^2 \sigma(r_0)^2 \omega'''(r_0) \right] \Big). \quad (3.14)$$

In principle, it is possible to compute the correction to the viscosity based only on the near horizon solution as demonstrated in [30]. However, since we already required the full solution in section II, we may just use the metric corrections given explicitly in appendix B. The leading order expressions are given in eq. (2.28). Substitution in (3.14) then yields the shear viscosity in terms of the charge and the horizon radius:

$$\eta = \frac{r_0^3}{16\pi G} \left[1 - 4b_1 (10 + \tilde{q}^2) - 4b_2 (2 - \tilde{q}^2) - 8b_3 (1 + \tilde{q}^2) - \frac{8}{3} b_1^2 (97\tilde{q}^4 - 172\tilde{q}^2 + 100) \right. \\ \left. - \frac{8}{3} b_3^2 (19\tilde{q}^4 + 32\tilde{q}^2 + 4) - \frac{8}{3} b_2 b_3 (7\tilde{q}^4 - 64\tilde{q}^2 + 28) - \frac{16}{3} b_1 b_3 (41\tilde{q}^4 - 2\tilde{q}^2 + 20) \right. \\ \left. - \frac{8}{3} b_1 b_2 (17\tilde{q}^4 - 140\tilde{q}^2 + 68) - 72c_1 (5\tilde{q}^4 - 12\tilde{q}^2 + 4) - 6c_2 (61\tilde{q}^4 - 100\tilde{q}^2 + 28) \right], \quad (3.15)$$

where $\tilde{q} \equiv q/r_0^3$ is the dimensionless charge parameter.

B. The Entropy Density

For the computation of the entropy density, we use Wald's formula for Noether charge density [48, 54]. Wald's formula is consistent with the first law of thermodynamics and therefore also with the Euclidean approach. The Noether charge entropy is given in the form of an integral over fields on a spatial section of the horizon \mathcal{H} . For theories without derivatives of the Riemann tensor, Wald's formula takes the following form [55]:

$$S_{BH} = -2\pi \int_{\mathcal{H}} \frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \sqrt{g_{\mathcal{H}}} d\Omega_{d-2} , \quad (3.16)$$

where the action of the d -dimensional theory is

$$I = \int d^d x \sqrt{-g} L , \quad (3.17)$$

and $\epsilon_{\mu\nu}$ is the binormal to the spatial section of the horizon \mathcal{H} , i.e. the volume element orthogonal to it. The binormal is defined by $\epsilon_{\mu\nu} = \nabla_{\mu} \chi_{\nu}$, where χ_{ν} is a Killing field normalized so that $\epsilon_{\mu\nu} \epsilon^{\mu\nu} = -2$. The volume element induced on \mathcal{H} is denoted $\sqrt{g_{\mathcal{H}}} d\Omega_{d-2}$. For black branes of the form (2.26), the integration in the spatial directions x, y, z gives an infinite factor. Therefore we consider only the entropy density s in those directions. Substitution of the corrected solution which appears in appendix B into the expression given by Wald's formula gives us the entropy density as

$$s = \frac{r_0^3}{4G} \left[1 - 4b_1(10 + \tilde{q}^2) - 8b_3(1 + \tilde{q}^2) - \frac{8}{3}b_1^2(97\tilde{q}^4 - 172\tilde{q}^2 + 100) - \frac{8}{3}b_3^2(19\tilde{q}^4 + 32\tilde{q}^2 + 4) \right. \\ \left. - 16b_2(2b_1 + b_3)(\tilde{q}^2 - 2)^2 - \frac{16}{3}b_1b_3(41\tilde{q}^4 - 2\tilde{q}^2 + 20) + 12c_1(7\tilde{q}^2 - 2)^2 + 9c_2(\tilde{q}^2 - 2) \right] . \quad (3.18)$$

C. The Ratio

Combining (3.18) with the result for the shear viscosity (3.15) and rewriting the expressions with \bar{r}_0 using eq. (B10), we find that the shear viscosity to entropy density ratio is given by

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 - 4b_2(2 - \bar{q}^2) - \frac{24b_2^2\bar{q}^2(2 - \bar{q}^2)}{2 + 5\bar{q}^2} + \frac{8b_1b_2(2 - \bar{q}^2)(49\bar{q}^4 - 250\bar{q}^2 - 140)}{3(2 + 5\bar{q}^2)} \right. \\ \left. - \frac{8b_2b_3(2 - \bar{q}^2)(\bar{q}^4 + 50\bar{q}^2 + 28)}{3(2 + 5\bar{q}^2)} - 12c_1(79\bar{q}^4 - 100\bar{q}^2 + 28) - 3c_2(125\bar{q}^4 - 212\bar{q}^2 + 68) \right] . \quad (3.19)$$

In the extremal limit $\bar{q}^2 \rightarrow 2$, the ratio becomes

$$\frac{\eta}{s} = \frac{1}{4\pi} [1 - 432(4c_1 + c_2)] . \quad (3.20)$$

Note that all contributions from the four-derivative terms vanish at the extremal limit, while the six-derivative corrections survive.

IV. CONSTRAINTS FROM THE WGC AND THE KSS BOUND

In this section we analyze the results for M/Q and η/s to determine the conditions under which the WGC and the KSS bound are compatible. Given the number of parameters in the action (1.4), it is convenient to discuss various cases in turn.

A. Gauss-Bonnet

We first consider theories in which the six-derivative terms are turned off, as would be required by supersymmetry for example. As we have seen above, $O(\alpha'^2)$ corrections to M/Q and η/s still arise due to terms quadratic in the four-derivative couplings. We begin by choosing a theory with $b_1 = b_3 = c_1 = c_2 = 0$. This combination of

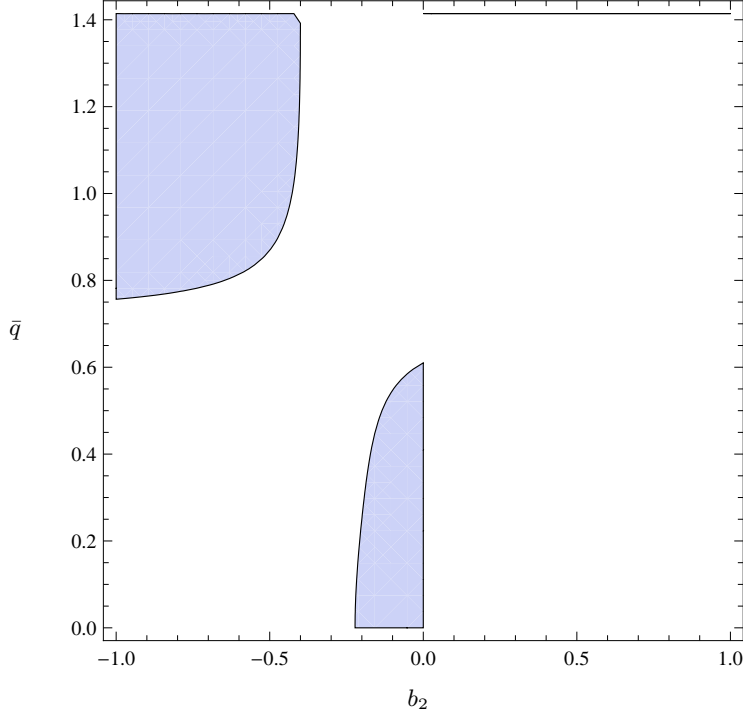


FIG. 2: Here we consider a theory with $b_1 = b_3 = c_1 = c_2 = 0$, i.e. we keep only the Gauss-Bonnet term in the gravitational action. The shaded regions represent points in the (b_2, \bar{q}) plane where both the WGC and the KSS bound are satisfied simultaneously. Note that the disconnected line segment in the upper right corner for $b_2 > 0$ corresponds to satisfaction of both constraints exactly at extremality. Since this case is disconnected from the non-extremal region and thus non-generic, it is not expected to occur physically.

the curvature-squared Gauss-Bonnet term and the absence of curvature-cubed terms arises, for example, in the low energy effective action of the heterotic string (see e.g. [56]).

At extremality, it follows from (2.37) and (3.20) that

$$\frac{M}{Q} = \left(\frac{M}{Q} \right)_0 \left(1 - b_2 - \frac{5}{2} b_2^2 \right), \quad \frac{\eta}{s} = \frac{1}{4\pi}. \quad (4.1)$$

The requirement that the α' corrections reduce M/Q implies $b_2 < -2/5$ or $b_2 > 0$. In fact, both bounds are satisfied in this range, as the KSS bound clearly holds for any b_2 .

Since we expect that the WGC should hold also for a neighborhood of the extremal limit, we now analyze the constraints for non-extremal black holes. The result (2.35) implies that for non-extremal black holes near extremality, the WGC is satisfied for $b_2 > 0$ or

$$b_2 < -\frac{2(5\bar{q}^2 + 2)^2(\bar{q}^4 + 5\bar{q}^2 - 2)}{9(15\bar{q}^8 + 69\bar{q}^6 + 62\bar{q}^4 - 36\bar{q}^2 - 8)} \leq -\frac{2}{5}. \quad (4.2)$$

The result (3.19) shows that the KSS bound holds (for all \bar{q}) when $b_2 < 0$, so near extremality both bounds are satisfied when (4.2) holds. The regions in the (b_2, \bar{q}) plane where the two bounds are compatible are plotted in Figure 2. Note that for the heterotic string theory effective action in particular, the Gauss-Bonnet coupling is positive [56], so in this case both bounds cannot be satisfied away from the extremal limit. However, if we instead consider the viscosity bound of $4/25\pi$ [29], the region of “good” theories is enlarged and does include cases with $b_2 > 0$ (see Figure 3).

B. Weyl-Tensor-Squared

As a second example of a four-derivative theory, we consider the Weyl-tensor-squared theory given by setting $b_1 = -5b_2/6, b_3 = 8b_2/3, c_1 = c_2 = 0$. A term of this form is present in higher derivative corrections to $\mathcal{N} = 2, d = 5$ gauged

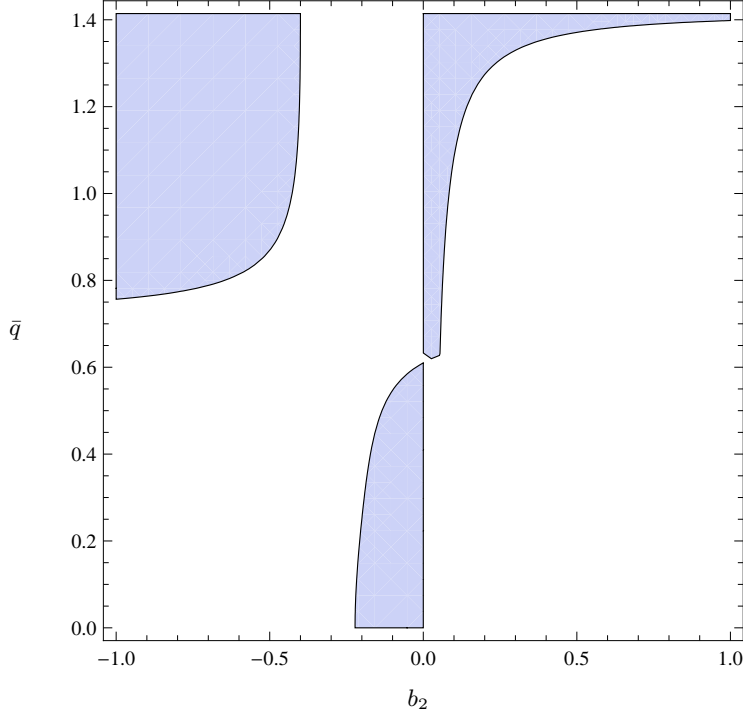


FIG. 3: Here we again consider only the Gauss-Bonnet correction, but instead relax the lower bound on η/s to be $4/25\pi$. The shaded regions represent points in the (b_2, \bar{q}) plane where both the WGC and the (improved) viscosity bound are satisfied simultaneously. Compared to Figure 2, there is now a nontrivial region near extremality for $0 < b_2 \ll 1$ where both constraints hold.

supergravity [8]. Such supergravity theories may arise from compactifying type IIB string theory on $AdS_5 \times X^5$, where X^5 is a Sasaki-Einstein manifold. According to the AdS/CFT correspondence, this theory is dual to $\mathcal{N} = 1, d = 4$ Super Yang Mills.

At extremality, it follows from (2.37) and (3.20) that

$$\frac{M}{Q} = \left(\frac{M}{Q} \right)_0 \left(1 - 3b_2 - \frac{152}{7}b_2^2 \right), \quad \frac{\eta}{s} = \frac{1}{4\pi}. \quad (4.3)$$

The requirement that the α' corrections reduce M/Q implies $b_2 < -21/52 \approx -0.14$ or $b_2 > 0$. In fact, both bounds are satisfied in this range, as the KSS bound clearly holds for any b_2 .

For general \bar{q} in the region near extremality, the WGC is satisfied for $b_2 > 0$ or

$$b_2 < -\frac{7(5\bar{q}^2 + 2)^2(19\bar{q}^6 - 82\bar{q}^4 - 8\bar{q}^2 + 48)}{4(1755\bar{q}^{12} - 3690\bar{q}^{10} - 10688\bar{q}^8 - 6536\bar{q}^6 - 10080\bar{q}^4 + 3136\bar{q}^2 + 448)} < 0. \quad (4.4)$$

The KSS bound is satisfied when $b_2 < 0$ or

$$b_2 > -\frac{5\bar{q}^2 + 2}{29\bar{q}^4 - 44\bar{q}^2 - 28} > 0. \quad (4.5)$$

Thus, near extremality both bounds are satisfied when (4.4) or (4.5) holds. The regions in the (b_2, \bar{q}) plane where the two bounds are compatible are plotted in Figure 4. For b_2 sufficiently small that the $O(b_2^2)$ corrections can be neglected, the two bounds are incompatible for non-extremal cases, as pointed out in [5]. However, we see that for $b_2 \lesssim -0.14$ the contribution from the $O(b_2^2)$ correction allows both bounds to be satisfied simultaneously.

For Weyl-tensor-squared theories, the behavior when the viscosity bound is relaxed to $4/25\pi$ is qualitatively similar to that discussed in the previous subsection.

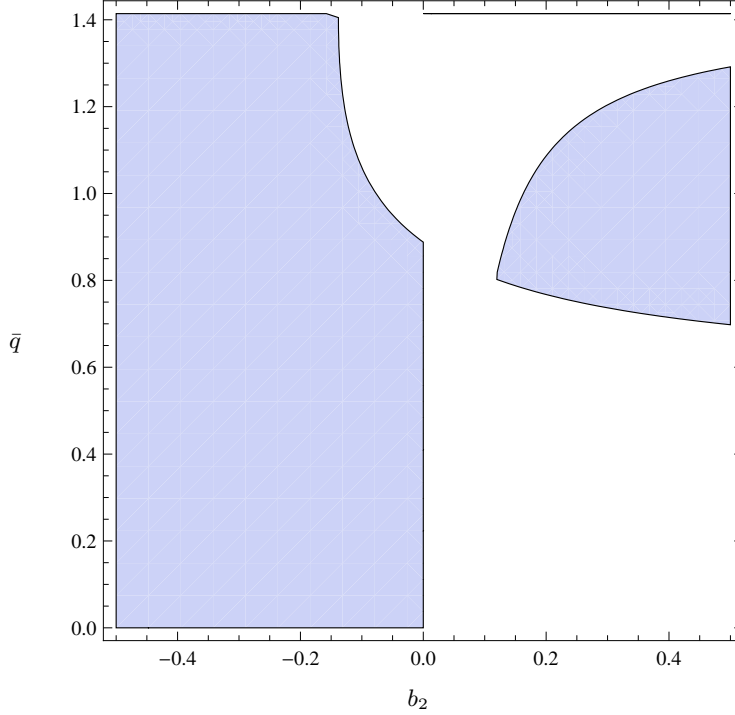


FIG. 4: Here we consider a theory with $b_1 = -5b_2/6$, $b_3 = 8b_2/3$, $c_1 = c_2 = 0$, which corresponds to keeping only the Weyl-tensor-squared term in the gravitational action. The shaded regions represent points in the (b_2, \bar{q}) plane where both the WGC and the KSS bound are satisfied simultaneously. See Figure 2 for comments about the disconnected line segment in the upper right corner.

C. Six-Derivatives

We now consider theories in which the first corrections to the effective action involve six-derivatives, i.e., $b_1 = b_2 = b_3 = 0$. Such theories may serve as toy models for non-supersymmetric string theory compactifications, which may in turn be dual to CFTs with broken supersymmetry [32]. Since we do not know in general whether c_1 and c_2 are related, we will consider all possibilities.

First suppose $c_2 = 0$. Then the WGC implies that $c_1 < 0$ for $0.73 \lesssim |\bar{q}| \leq \sqrt{2}$ and $c_1 > 0$ for $0 < |\bar{q}| \lesssim 0.73$. Imposing the KSS bound requires $c_1 < 0$ for $0.92 \lesssim |\bar{q}| \leq \sqrt{2}$ or $0 < |\bar{q}| \lesssim 0.65$, and $c_1 > 0$ for $0.65 \lesssim |\bar{q}| \lesssim 0.92$. Thus, both bounds can be satisfied when $0.92 \lesssim |\bar{q}| \leq \sqrt{2}$ with $c_1 < 0$ (which includes the extremal case) and $0.65 \lesssim |\bar{q}| \lesssim 0.73$ with $c_1 > 0$.

Now suppose $c_1 = 0$. Then the WGC is satisfied for all $0 < |\bar{q}| \leq \sqrt{2}$ when $c_2 < 0$. This condition is compatible with the KSS bound when $1.12 \lesssim |\bar{q}| \leq \sqrt{2}$ (which includes the extremal case) and $0 < |\bar{q}| \lesssim 0.66$.

Finally, we consider both $c_1, c_2 \neq 0$ and focus on the region near extremality. We have

$$\frac{M}{Q} = \left(\frac{M}{Q} \right)_0 (1 + \alpha_1(\bar{q})c_1 + \alpha_2(\bar{q})c_2 + \dots), \quad \frac{\eta}{s} = \frac{1}{4\pi} (1 + \beta_1(\bar{q})c_1 + \beta_2(\bar{q})c_2 + \dots), \quad (4.6)$$

where the functions $\alpha_i(\bar{q}), \beta_i(\bar{q})$ may read off from (2.35), (3.19) and near extremality satisfy $\alpha_i(\bar{q}) > 0, \beta_i(\bar{q}) < 0$. In contrast to the four-derivative theories considered above, here we observe that $\eta/s \neq 1/4\pi$ at the extremal limit. Note also that even when the curvature squared terms are also present, the sign of the first correction to η/s at extremality is determined by the curvature-cubed terms.

First suppose $c_1 < 0$. Then both bounds are satisfied for any $c_2 < 0$. For $c_2 > 0$, both bounds can hold simultaneously if $c_2 < \min(|\alpha_1 c_1 / \alpha_2|, |\beta_1 c_1 / \beta_2|)$, where $\min(a, b)$ ($\max(a, b)$) denotes the smaller (larger) of a, b .

Now suppose $c_1 > 0$. Then the WGC implies that we must have $c_2 < 0$. Both bounds can be satisfied if $|c_2| > \max(|\alpha_1 c_1 / \alpha_2|, |\beta_1 c_1 / \beta_2|)$.

The regions in the (c_1, c_2) plane where both bounds are satisfied are plotted for several values of \bar{q} in Figure 5. For these theories, the plots do not change substantially if the viscosity bound is relaxed to $4/25\pi$. For the case of black branes with spherical horizons, see appendix C.

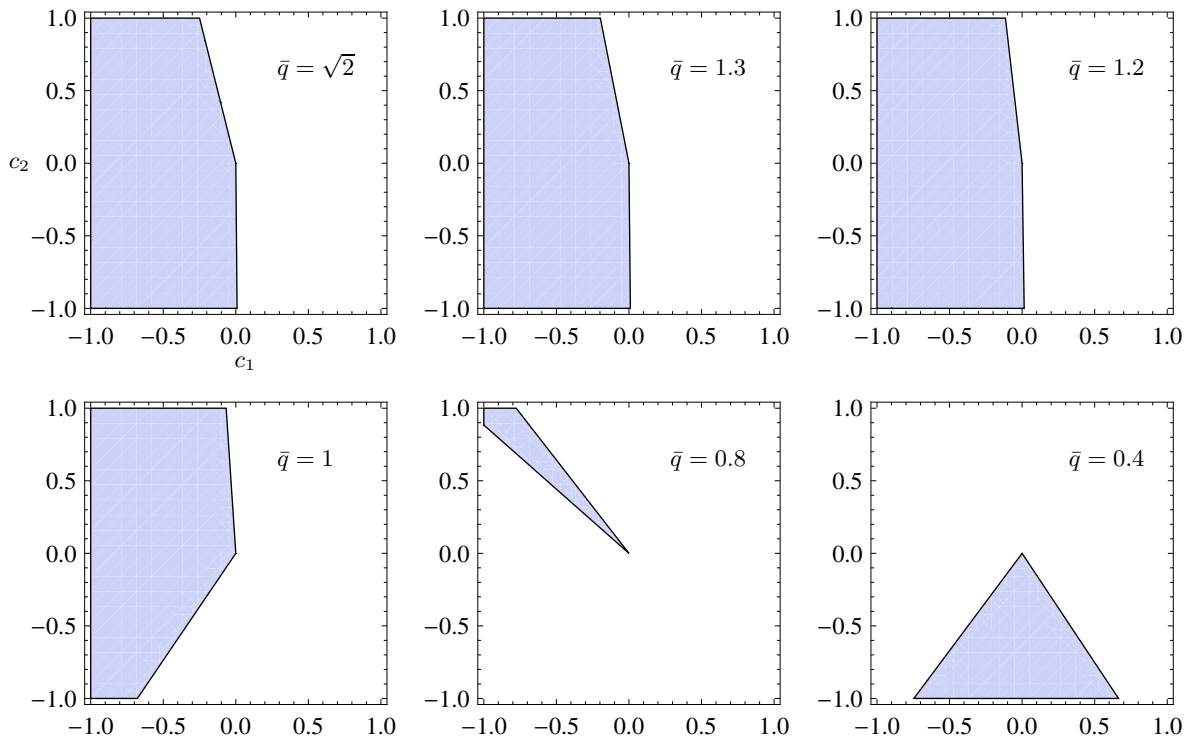


FIG. 5: Here we consider a theory with $b_1 = b_2 = b_3 = 0$, i.e. we keep only the six-derivative terms in the gravitational action. The shaded regions represent points in the (c_1, c_2) plane (for various values of \bar{q}) where both the WGC and the KSS bound are satisfied simultaneously.

V. CONCLUSIONS

In this work, we examined the WGC and KSS bound for four-derivative and six-derivative corrections to charged AdS_5 black branes. The WGC states that higher derivative corrections decrease the mass to charge ratio of extremal black holes. The KSS bound is a lower bound on the shear viscosity to entropy density ratio, $\eta/s \geq 1/4\pi$. In particular, we studied the interplay of these constraints with leading and next-to-leading corrections for a family of toy-models (1.4). Such constraints on effective theories might help to distinguish which theories can be UV completed. First, we calculated the higher derivative corrections to the mass density in an AdS background using the covariant ADT method. For the same type of branes, we then calculated corrections to the shear viscosity (using holographic methods) and the entropy density (using Wald’s formula). Using these results, we analyzed the constraints on the mass to charge ratio (WGC) and the shear viscosity to entropy density ratio (KSS bound). This analysis of the two constraints required comparison of various thermodynamical quantities and their ratios (energy, charge/chemical potential, viscosity and entropy) between uncorrected and higher-derivative corrected branes. In order to make a meaningful comparison, it is important to consider quantities at the same temperature (e.g. the canonical ensemble), especially when we compare quantities in different theories. For this purpose, we rewrote the ratios M/Q and η/s using a parameter \bar{r}_0 (defined in (2.34)), which corresponds to keeping the temperature unchanged.

One of our main conclusions is that six-derivative corrections in general behave differently than four-derivative corrections. As noted in [5], for typical examples of four-derivative corrections (e.g. Gauss-Bonnet) the sign of the correction to M/Q is opposite relative to the sign of the correction to η/s . Thus, the constraints are mutually exclusive, which might suggest that the theory cannot be UV completed. In contrast, if we consider a particular coefficient of the six-derivative corrections (say c_1 or c_2 in (1.4)), we find that both constraints require the same sign of the coefficient. Hence, in the sense of the WGC and KSS bound, six-derivative terms tend to be “good” corrections. It is interesting to note that the “good” behavior of the six-derivative terms might be related to the fact that for those terms the shear viscosity to entropy density ratio at extremality does not vanish. A similar behavior was observed for the $R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$ term in four-derivative actions [9]. Note that this “good” behavior does not depend on the exact value of the lower bound on η/s . We also found that working to order α'^2 in theories with four-derivative corrections

(e.g. Gauss-Bonnet or Weyl-squared) introduced a wider possibility in the parameter space to satisfy the constraints.

In the context of this work, we found that the ADT method is a convenient way to calculate the energy. The procedure is based on linearizing the equations of motion and does not require explicit expressions for boundary counterterms in the action (note that we also avoided counterterms in the viscosity calculation following [7]). By construction, the background AdS space automatically has vanishing energy. As seen above, the effective stress tensor (2.13) for the theory (1.4) maintained precisely the same form as in four-derivative gravity. In particular, the six-derivative terms only changed the coefficients α_i . It would be interesting to understand if this result holds more generally, say for other curvature-cubed terms or even higher derivative terms. One might also try to determine if the form of (2.13) is in fact the unique tensor with the desired properties. If so, it may be possible to find an even faster general method to extract the corrections to the coefficients directly from the action.

The action (1.4) contains two six-derivative corrections that (up to field redefinitions) represent all possible six-derivative terms when a gauge field is not included. When a gauge field is added, there are many more types of possible correction terms [7]. We considered only a limited family of corrections in order to learn about the possibilities opened by including higher than quadratic derivative terms. As a future direction, it would be interesting to consider the most general action with six-derivative corrections including a gauge field. Note that the effective stress-tensor (2.13) computation does not depend on the matter part of the action and therefore is already given in the most general form for six-derivative corrections.

Finally, our expressions for M/Q , η/s were given as perturbative expansions in the couplings b_i, c_i , but it would also be interesting to rewrite these results in terms of physical CFT parameters, like the central charges c, a or the flux coefficient t_4 . One could then add to the analysis the constraints on these parameters that arise from various CFT considerations, such as positivity of the energy flux [33, 57].

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Appendix A: The Linearized Equation of Motion

In this appendix we collect some results that are useful for obtaining the final expression for the energy (2.21).

To find the effective stress-energy tensor, we wish to linearize the equations of motion (2.8) about a pure AdS background satisfying

$$\bar{R}_{\mu\rho\nu\sigma} = k(\bar{g}_{\mu\nu}\bar{g}_{\rho\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\rho\nu}), \quad \bar{R}_{\mu\nu} = (d-1)k\bar{g}_{\mu\nu}, \quad \bar{R} = d(d-1)k, \quad (\text{A1})$$

where

$$k \equiv \frac{2\Lambda_{eff}}{(d-1)(d-2)}. \quad (\text{A2})$$

Here the cosmological constant Λ_{eff} satisfies (2.12), whose perturbative solution is

$$\begin{aligned} \Lambda_{eff} = & \Lambda - \frac{2(d-4)\Lambda^2(db_1 + b_3)}{(d-2)^2} - \frac{2(d-4)(d-3)\Lambda^2 b_2}{(d-1)(d-2)} - \frac{16(d-6)\Lambda^3 c_1}{(d-2)^3(d-1)^2} \\ & - \frac{4(d-6)\Lambda^3 c_2}{(d-1)^2(d-2)^2} + \frac{8d^2(d-4)^2\Lambda^3 b_1^2}{(d-2)^4} + \frac{16d(d-4)^2\Lambda^3 b_1 b_3}{(d-2)^4} + \frac{8(d-4)^2\Lambda^3 b_3^2}{(d-2)^4} \\ & + \frac{16(d-4)^2(d-3)\Lambda^3 b_2(db_1 + b_3)}{(d-2)^3(d-1)} + \frac{8(d-4)^2(d-3)^2\Lambda^3 b_2^2}{(d-1)^2(d-2)^2} + \dots \end{aligned} \quad (\text{A3})$$

To perform the relevant linearizations, it is useful to recall that for a decomposition of the metric as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, the linearized Riemann tensor is

$$(R^\rho{}_{\mu\lambda\nu})_L = \bar{\nabla}_\lambda \Gamma_{\mu\nu}^\rho - \bar{\nabla}_\nu \Gamma_{\lambda\mu}^\rho, \quad (\text{A4})$$

where

$$\Gamma_{\mu\nu}^\rho = \frac{1}{2}\bar{g}^{\rho\sigma} (\bar{\nabla}_\mu h_{\nu\sigma} + \bar{\nabla}_\nu h_{\mu\sigma} - \bar{\nabla}_\sigma h_{\mu\nu}) . \quad (\text{A5})$$

The linearized Ricci tensor is

$$R_{\mu\nu}^L = \frac{1}{2}(-\bar{\square}h_{\mu\nu} + \bar{\nabla}^\lambda \bar{\nabla}_\nu h_{\mu\lambda} + \bar{\nabla}^\lambda \bar{\nabla}_\mu h_{\nu\lambda} - \bar{\nabla}_\mu \bar{\nabla}_\nu h) \quad (\text{A6})$$

and the linearized Ricci scalar is

$$R_L = (g^{\mu\nu} R_{\mu\nu})_L = -\bar{\square}h + \bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} - \bar{R}^{\mu\nu} h_{\mu\nu} . \quad (\text{A7})$$

It is then straightforward (but lengthy) to show that the linearizations of the six-derivative terms in (2.8) are

$$\begin{aligned} (\nabla_\kappa \nabla_\rho (R_\mu^{\rho\lambda\tau} R_{\lambda\tau}{}^\kappa{}_\nu))_L &= -4k(\bar{\square}R_{\mu\nu}^L - \frac{1}{2}\bar{\nabla}_\mu \bar{\nabla}_\nu R_L) + 4dk^2 R_{\mu\nu}^L - 4k^2 R_L \bar{g}_{\mu\nu} \\ &\quad + 4k^2(d-1)\bar{\square}h_{\mu\nu} - 4k^3 d(d-1)h_{\mu\nu} \end{aligned} \quad (\text{A8})$$

$$(R_{\sigma\mu\lambda\rho} R^{\lambda\rho\alpha\beta} R^\sigma{}_{\nu\alpha\beta})_L = 12k^2 R_{\mu\nu}^L - 8k^3(d-1)h_{\mu\nu} \quad (\text{A9})$$

$$\begin{aligned} (\nabla_\kappa \nabla_\rho (R_{\mu\tau\lambda\nu} R^{\rho\tau\lambda\kappa}))_L &= -k\bar{\square}(R_{\mu\nu}^L - \frac{1}{2}\bar{g}_{\mu\nu} R_L) + k\bar{\nabla}_\mu \bar{\nabla}_\nu R_L + 2dk^2 R_{\mu\nu}^L - 2k^2 R_L \bar{g}_{\mu\nu} \\ &\quad + (d-1)k^2 \bar{\square}h_{\mu\nu} - 2k^3 d(d-1)h_{\mu\nu} \end{aligned} \quad (\text{A10})$$

$$(\nabla_\kappa \nabla_\rho (R_\mu^{\tau\lambda\kappa} R^\rho{}_{\tau\lambda\nu}))_L = k\bar{\nabla}_\mu \bar{\nabla}_\nu R_L + dk^2 R_{\mu\nu}^L - k^2 R_L \bar{g}_{\mu\nu} - k^3 d(d-1)h_{\mu\nu} \quad (\text{A11})$$

$$(R^\beta{}_{\mu\rho\sigma} R^{\rho\tau}{}_{\lambda\beta} R^\sigma{}_\tau{}^\lambda{}_\nu)_L = 2(d-3)k^2 R_{\mu\nu}^L + k^2 \bar{g}_{\mu\nu} R_L + k^3(d-1)(4-d)h_{\mu\nu} . \quad (\text{A12})$$

Combining these results with those of [37] yields (2.13).

Appendix B: The Corrected Metric

We substitute the ansatz (2.26) in the action (1.4) and obtain 3 equations of motion by variation with respect to $\omega(r)$, $\sigma(r)$ and $\gamma(r)$. Using the zeroth order solution, we can solve the equations of motion perturbatively in α' . It is convenient to work in a scheme where the horizon radius is fixed at $r = r_0$, i.e. not corrected by the higher derivative contributions. This can be achieved by choosing the integration constants appropriately when solving the gravitational field equations at each order in α' . The cosmological constant gets corrected as given in (A3). We also choose integration constants when solving Maxwell's equations so that the charge is not corrected and remains as in the zeroth order. The solution is

$$\begin{aligned} \omega &= \omega_{(0)} + \omega_{(1)} + \omega_{(2)} \quad (\text{B1}) \\ \omega_{(1)} &= \frac{b_1}{6r^{10}r_0^8} [40r^8 r_0^8 (r^4 - r_0^4) - 16q^2 r^2 r_0^6 (13r^6 - 15r^4 r_0^2 + 2r_0^6) + q^4 (47r_0^8 - 32r^2 r_0^6 - 15r^8)] \\ &\quad + \frac{2b_2}{r^{10}r_0^4} [q^2 r^2 - (q^2 + r^6)r_0^2 + r^2 r_0^6]^2 + \frac{b_3}{6r^{10}r_0^8} [8r^8 r_0^8 (r^4 - r_0^4) + 8q^2 r^2 r_0^6 (2r^6 - 3r^4 r_0^2 + r_0^6) \\ &\quad + q^4 (13r_0^8 + 8r^2 r_0^6 - 21r^8)] \end{aligned}$$

$$\begin{aligned}
\omega_{(2)} = & -\frac{8b_2^2}{r^{16}r_0^6} [q^2 r^2 - (q^2 + r^6)r_0^2 + r^2 r_0^6]^3 - \frac{4b_1^2}{9r^{16}r_0^{14}} [200r^{14}r_0^{14}(r_0^4 - r^4) + 24q^2 r^4 r_0^{12}(209r^{10} \\
& - 175r^8 r_0^2 + 62r^4 r_0^6 - 96r_0^{10}) + q^4 r^2 r_0^6 (6544r_0^{12} - 4608r^2 r_0^{10} - 2773r^4 r_0^8 + 1824r^6 r_0^6 - 987r^{12}) \\
& + q^6 (398r^{14} + 30r^8 r_0^6 - 2304r^4 r_0^{10} + 6544r^2 r_0^{12} - 4668r_0^{14})] + \frac{2b_3^2}{63r^{16}r_0^{14}} [112r^{14}r_0^{14}(r^4 - r_0^4) \\
& + 168q^2 r^4 r_0^{12}(26r^{10} + 2r^8 r_0^2 - 67r^4 r_0^6 + 39r_0^{10}) + q^6 (7977r_0^{14} - 1264r^{14} + 147r^8 r_0^6 + 6552r^4 r_0^{10} \\
& - 13412r^2 r_0^{12}) - 28q^4 r^2 r_0^6 (479r_0^{12} - 468r^2 r_0^{10} - 461r^4 r_0^8 + 408r^6 r_0^6 + 42r^{12})] \\
& - \frac{2b_1 b_3}{63r^{16}r_0^{14}} [1120r^{14}r_0^{14}(r_0^4 - r^4) + 840q^2 r^4 r_0^{12}(8r^{10} + 11r^8 r_0^2 + 5r^4 r_0^6 - 24r_0^{10}) - 14q^4 r^2 r_0^6 (69r^{12} \\
& - 168r^6 r_0^6 + 1309r^4 r_0^8 + 2880r^2 r_0^{10} - 4090r_0^{12}) + q^6 (4972r^{14} + 483r^8 r_0^6 - 20160r^4 r_0^{10} + 57260r^2 r_0^{12} \\
& - 42555r_0^{14})] + \frac{2b_2 b_3}{21r^{16}r_0^{14}} [112r^{10}r_0^{14}(r^8 + 3r^4 r_0^4 - 4r_0^8) - 28q^2 r^4 r_0^{12}(54r^{10} + 13r^8 r_0^2 + 38r^6 r_0^4 \\
& - 96r^4 r_0^6 - 9r_0^{10}) + 7q^4 r^2 r_0^6 (69r^{12} - 67r^8 r_0^4 + 408r^6 r_0^6 - 289r^4 r_0^8 + 72r^2 r_0^{10} - 193r_0^{12}) \\
& + q^6 (1207r_0^{14} - 1351r^2 r_0^{12} + 252r^4 r_0^{10} - 147r^8 r_0^6 + 147r^{10} r_0^4 - 108r^{14})] \\
& + \frac{2b_1 b_2}{15r^{16}r_0^{14}} [8q^2 r^4 (r_0^2 - r^2) r_0^{12} (567r^8 + 167r^6 r_0^2 - 138r^4 r_0^4 + 219r^2 r_0^6 + 399r_0^8) + q^4 r^2 r_0^6 (843r^{12} \\
& + 1715r^8 r_0^4 - 3696r^6 r_0^6 + 785r^4 r_0^8 + 6384r^2 r_0^{10} - 6031r_0^{12}) + q^6 (3475r_0^{14} - 6031r^2 r_0^{12} + 3192r^4 r_0^{10} \\
& - 480r^6 r_0^8 - 75r^8 r_0^6 + 75r^{10} r_0^4 - 156r^{14}) + 80r^6 r_0^{14} (5r^{12} - 9r^8 r_0^4 + 10r^4 r_0^8 - 6r_0^{12})] \\
& + \frac{c_1}{105r^{16}r_0^{14}} [84q^2 r^4 (r^2 - r_0^2) r_0^{12} (554r^8 + 959r^6 r_0^2 + 1289r^4 r_0^4 - 117r^2 r_0^6 - 147r_0^8) + 140r^6 r_0^{14} (r^{12} \\
& - 94r^8 r_0^4 + 99r^4 r_0^8 - 6r_0^{12}) - 21q^4 r^2 r_0^6 (4853r^{12} - 660r^8 r_0^4 + 5624r^6 r_0^6 - 9455r^4 r_0^8 - 1176r^2 r_0^{10} + 814r_0^{12}) \\
& + 2q^6 (17813r^{14} - 420r^6 r_0^8 + 6174r^4 r_0^{10} - 8547r^2 r_0^{12} - 15020r_0^{14})] + \frac{c_2}{140r^{16}r_0^{14}} [84q^2 r^4 (r^2 - r_0^2) r_0^{12} (112r^8 \\
& + 17r^6 r_0^2 + 107r^4 r_0^4 - 91r^2 r_0^6 - 171r_0^8) + 140r^6 r_0^{14} (r^{12} - 12r^8 r_0^4 + 27r^4 r_0^8 - 16r_0^{12}) - 63q^4 r^2 r_0^6 (53r^{12} \\
& - 60r^8 r_0^4 + 264r^6 r_0^6 - 215r^4 r_0^8 - 456r^2 r_0^{10} + 414r_0^{12}) + 2q^6 (279r^{14} - 1120r^6 r_0^8 + 7182r^4 r_0^{10} - 13041r^2 r_0^{12} \\
& + 6700r_0^{14})]
\end{aligned}
\tag{B2}$$

$$\sigma = 1 + \sigma_{(1)} + \sigma_{(2)} \tag{B3}$$

$$\sigma_{(1)} = \frac{4q^2}{3r^6} (7b_1 + 5b_3) \tag{B4}$$

$$\begin{aligned}
\sigma_{(2)} = & -\frac{8b_1^2 q^2}{9r^{12}r_0^2} (3168(q^2 + r_0^6)r^2 - 7(719q^2 + 176r^6)r_0^2) - \frac{16b_3^2 q^2}{9r^{12}r_0^2} (18(q^2 + r_0^6)r^2 - 70r^6 r_0^2 - 103q^2 r_0^2) \\
& + \frac{8b_2 b_3}{3r^{12}r_0^4} (30r^4 r_0^{12} + 20q^2 r^2 r_0^4 (r^4 + 3r^2 r_0^2 - 9r_0^4) + q^4 (30r^4 - 180r^2 r_0^2 + 193r_0^4)) \\
& - \frac{8b_1 b_3 q^2}{9r^{12}r_0^2} (1980r^2 r_0^6 - 1076r^6 r_0^2 + 5q^2 (396r^2 - 673r_0^2)) \\
& + \frac{8b_2 b_1}{15r^{12}r_0^4} (270r^4 r_0^{12} + q^4 (270r^4 - 1692r^2 r_0^2 + 1855r_0^4) + 4q^2 (35r^6 r_0^4 + 135r^4 r_0^6 - 423r^2 r_0^8)) \\
& - \frac{2c_1}{5r^{12}r_0^4} (390r^4 r_0^{12} + q^4 (390r^4 - 3564r^2 r_0^2 + 7705r_0^4) + 4q^2 (65r^6 r_0^4 + 195r^4 r_0^6 - 891r^2 r_0^8)) \\
& + \frac{3c_2}{10r^{12}r_0^4} (30r^4 r_0^{12} + q^4 (30r^4 - 108r^2 r_0^2 + 85r_0^4) + 4q^2 (5r^6 r_0^4 + 15r^4 r_0^6 - 27r^2 r_0^8))
\end{aligned}
\tag{B5}$$

$$\gamma = \gamma_{(0)} + \gamma_{(1)} + \gamma_{(2)} \quad (\text{B6})$$

$$\frac{\gamma_{(1)}}{\gamma_{(0)}} = \frac{q^2}{3r^6} (7b_1 + 5b_3) \quad (\text{B7})$$

$$\begin{aligned} \frac{\gamma_{(2)}}{\gamma_{(0)}} = & -\frac{8b_1^2 q^2}{9r^{12}r_0^2} (528r^2(r_0^6 + q^2) - 308r^6r_0^2 - 719q^2r_0^2) - \frac{8b_3^2 q^2}{9r^{12}r_0^2} (42r^2(r_0^6 + q^2) - 245r^6r_0^2 - 206q^2r_0^2) \\ & - \frac{8b_1b_3 q^2}{63r^{12}r_0^2} (2310r^2r_0^6 - 1883r^6r_0^2 + 5q^2(462r^2 - 673r_0^2)) \\ & + \frac{8b_1b_2}{15r^{12}r_0^4} (54r^4(r_0^{12} + q^4) + q^4r_0^2(265r_0^2 - 282r^2) + q^2r_0^4(35r^6 + 108r^4r_0^2 - 282r^2r_0^4)) \\ & + \frac{8b_2b_3}{21r^{12}r_0^4} (42r^4(r_0^{12} + q^4) + q^4r_0^2(193r_0^2 - 210r^2) + 7q^2r_0^4(5r^6 + 12r^4r_0^2 - 30r^2r_0^4)) \\ & + \frac{2c_1}{35r^{12}r_0^4} (546r^4(r_0^{12} + q^4) + q^4r_0^2(7705r_0^2 - 4158r^2) + 7q^2r_0^4(65r^6 + 156r^4r_0^2 - 594r^2r_0^4)) \\ & + \frac{3c_2}{70r^{12}r_0^4} (42r^4(r_0^{12} + q^4) + q^4r_0^2(85r_0^2 - 126r^2) + 7q^2r_0^4(5r^6 + 12r^4r_0^2 - 18r^2r_0^4)). \end{aligned} \quad (\text{B8})$$

Substituting in eq. (2.30), we get the temperature of the corrected solution as:

$$\begin{aligned} T = & \frac{r_0}{\pi} \left[1 - \frac{\tilde{q}^2}{2} - \frac{1}{3} (b_1(9\tilde{q}^4 + 64\tilde{q}^2 - 20) + b_3(9\tilde{q}^4 + 20\tilde{q}^2 - 4)) + \frac{c_1}{15} (4583\tilde{q}^6 - 138\tilde{q}^4 - 1086\tilde{q}^2 + 620) \right. \\ & + \frac{3c_2}{20} (\tilde{q}^2 - 2)(3\tilde{q}^4 - 12\tilde{q}^2 + 10) - \frac{4b_2}{15} (\tilde{q}^2 - 2)(5b_3(9\tilde{q}^4 - 12\tilde{q}^2 - 10) + b_1(93\tilde{q}^4 - 132\tilde{q}^2 - 130)) \\ & - \frac{4b_1^2}{9} (493\tilde{q}^6 - 84\tilde{q}^4 + 1896\tilde{q}^2 - 200) - \frac{4b_3^2}{9} (91\tilde{q}^6 + 222\tilde{q}^4 + 204\tilde{q}^2 - 8) \\ & \left. - \frac{8b_1b_3}{9} (212\tilde{q}^6 + 255\tilde{q}^4 + 570\tilde{q}^2 - 40) \right], \end{aligned} \quad (\text{B9})$$

where $\tilde{q} \equiv q/r_0^3$ is a dimensionless charge parameter. We write the CFT temperature $T_{CFT} \equiv \ell_{eff}T$ so that the higher derivative corrections are absorbed in a redefined parameter \bar{r}_0 :

$$\begin{aligned} r_0 = & \bar{r}_0 \left[1 + \frac{2}{15\bar{q}^2 + 6} (b_1(9\bar{q}^4 + 59\bar{q}^2 - 10) - 3b_2(\bar{q}^2 - 2) + b_3(9\bar{q}^4 + 19\bar{q}^2 - 2)) \right. \\ & + \frac{2b_2(\bar{q}^2 - 2)}{5(5\bar{q}^2 + 2)^3} (5b_3(300\bar{q}^8 - 85\bar{q}^6 - 621\bar{q}^4 - 284\bar{q}^2 - 60) + b_1(3100\bar{q}^8 - 1545\bar{q}^6 - 8769\bar{q}^4 - 3604\bar{q}^2 - 860)) \\ & + \frac{4b_1b_3}{9(5\bar{q}^2 + 2)^3} (14720\bar{q}^{10} + 2446\bar{q}^8 + 32263\bar{q}^6 + 20594\bar{q}^4 + 10228\bar{q}^2 - 200) \\ & + \frac{2b_1^2}{9(5\bar{q}^2 + 2)^3} (42820\bar{q}^{10} - 27874\bar{q}^8 + 69839\bar{q}^6 + 48106\bar{q}^4 + 33716\bar{q}^2 - 1000) \\ & + \frac{2b_3^2}{9(5\bar{q}^2 + 2)^3} (2620\bar{q}^{10} + 8366\bar{q}^8 + 20687\bar{q}^6 + 11290\bar{q}^4 + 3572\bar{q}^2 - 40) \\ & \left. - \frac{2c_1}{15(5\bar{q}^2 + 2)} (4583\bar{q}^6 - 138\bar{q}^4 - 1081\bar{q}^2 + 610) - \frac{c_2}{10(\bar{q}^2 + 2)} (q^2 - 2)(3q^2 - 7)(3q^2 - 5) \right], \end{aligned} \quad (\text{B10})$$

where we have set $\bar{q} \equiv q/\bar{r}_0^3$.

Appendix C: Mass to Charge Ratio for Spherical Horizons

In the text above, we discussed M/Q for planar black hole solutions. However, given that the original formulation of the WGC [1] applied to asymptotically flat extremal black holes, it is perhaps more appropriate to consider M/Q

for black holes with a spherical horizon [7]. This further allows us to check another aspect of the WGC, namely the prediction that the correction to the mass to charge ratio should become more negative for smaller extremal black holes.

It is straightforward to repeat the calculations of section II C for the case of spherical horizons, i.e. with a metric ansatz

$$ds^2 = -\omega(r)dt^2 + \frac{\sigma^2(r)}{\omega(r)}dr^2 + r^2 d\Omega_3^2, \quad (\text{C1})$$

$$(\text{C2})$$

where $d\Omega_3^2$ is the line element of the unit S^3 . In the absence of higher derivative corrections, the solution to the equations of motion is now given by

$$\omega_{(0)}(r) = 1 + r^2 - \frac{q^2 + r_0^4 + r_0^6}{r_0^2 r^2} + \frac{q^2}{r^4} \quad (\text{C3})$$

$$\sigma_{(0)}(r) = 1 \quad (\text{C4})$$

$$\gamma_{(0)}(r) = \frac{\sqrt{3}q}{r^2}. \quad (\text{C5})$$

The horizon is at $r = r_0$ and the extremal case is $q^2 = 2r_0^6(1 + \frac{1}{2r_0^2})$.

In the interest of brevity, we omit the full solution and the mass to charge ratio for general q when the higher derivative corrections are included. Instead we just give the result for the mass to charge ratio in the extremal case:

$$\begin{aligned} \frac{M}{Q} = & \left(\frac{M}{Q} \right)_0 \left(1 - \frac{264\bar{r}_0^4 + 284\bar{r}_0^2 + 1}{6\bar{r}_0^2(3\bar{r}_0^2 + 2)}b_1 - \frac{3\bar{r}_0^4 + 2\bar{r}_0^2 - 2}{\bar{r}_0^2(3\bar{r}_0^2 + 2)}b_2 - \frac{(8\bar{r}_0^2 + 1)(12\bar{r}_0^2 + 11)}{6\bar{r}_0^2(3\bar{r}_0^2 + 2)}b_3 \right. \\ & + \frac{31182\bar{r}_0^6 + 29392\bar{r}_0^4 + 8865\bar{r}_0^2 + 536}{105\bar{r}_0^4(3\bar{r}_0^2 + 2)}c_1 + \frac{366\bar{r}_0^6 + 136\bar{r}_0^4 - 765\bar{r}_0^2 - 152}{140\bar{r}_0^4(3\bar{r}_0^2 + 2)}c_2 - \frac{15\bar{r}_0^4 + 10\bar{r}_0^2 + 4}{2\bar{r}_0^2(3\bar{r}_0^2 + 2)}b_2^2 \\ & - \frac{103950\bar{r}_0^8 + 99306\bar{r}_0^6 + 23013\bar{r}_0^4 - 929\bar{r}_0^2 - 67}{45\bar{r}_0^4(3\bar{r}_0^2 + 1)(3\bar{r}_0^2 + 2)}b_1^2 - \frac{1752\bar{r}_0^6 + 52\bar{r}_0^4 - 541\bar{r}_0^2 - 96}{42\bar{r}_0^4(3\bar{r}_0^2 + 2)}b_2b_3 \\ & - \frac{19170\bar{r}_0^8 + 19710\bar{r}_0^6 + 5661\bar{r}_0^4 + 383\bar{r}_0^2 + 55}{63\bar{r}_0^4(3\bar{r}_0^2 + 1)(3\bar{r}_0^2 + 2)}b_3^2 - \frac{4128\bar{r}_0^6 + 1348\bar{r}_0^4 - 325\bar{r}_0^2 - 96}{30\bar{r}_0^4(3\bar{r}_0^2 + 2)}b_2b_1 \\ & \left. - \frac{2(50814\bar{r}_0^8 + 48870\bar{r}_0^6 + 11322\bar{r}_0^4 - 326\bar{r}_0^2 + 5)}{63\bar{r}_0^4(3\bar{r}_0^2 + 1)(3\bar{r}_0^2 + 2)}b_3b_1 + \dots \right) \quad (\text{C6}) \end{aligned}$$

where

$$\left(\frac{M}{Q} \right)_0 = \frac{\sqrt{3}(3\bar{r}_0^2 + 2)}{64\pi G\sqrt{2\bar{r}_0^2 + 1}}. \quad (\text{C7})$$

Similarly to section II C, the parameter \bar{r}_0 has been defined by fixing

$$T_{CFT} = \frac{\bar{r}_0}{\pi} \left(1 - \frac{q^2}{2\bar{r}_0^6} + \frac{1}{2\bar{r}_0^2} \right), \quad (\text{C8})$$

with the extremal case corresponding to $q^2 = 2\bar{r}_0^6(1 + \frac{1}{2\bar{r}_0^2})$. This reparametrization ensures that the uncorrected and corrected values of M/Q are compared at the same temperature. Note that (C6) reduces to the planar black hole result (2.37) in the limit $\bar{r}_0 \rightarrow \infty$.

In the case of spherical horizons, the analysis of the two constraints is qualitatively similar to the planar case until $\bar{r}_0 \lesssim 1$, at which point the behavior of M/Q may change. However, it is plausible that the exact black hole solution might not exist for \bar{r}_0 of order the AdS radius. Let us consider for example theories in which the first corrections to the effective action involve six-derivatives, i.e., $b_1 = b_2 = b_3 = 0$.

First suppose $c_2 = 0$. The coefficient of c_1 in (C6) is positive for all \bar{r}_0 , so the WGC requires $c_1 < 0$. Note that because of the factor \bar{r}_0^4 in the denominator, the correction becomes more negative for smaller \bar{r}_0 as expected. When $c_1 < 0$, the KSS bound is also satisfied.

Now suppose $c_1 = 0$. The coefficient of c_2 in (C6) is positive for large \bar{r}_0 , but becomes negative for $\bar{r}_0 \lesssim 1.17$. The KSS bound meanwhile requires $c_2 < 0$, so both bounds can be satisfied only when $\bar{r}_0 \gtrsim 1.17$ and $c_2 < 0$. Once again, the correction to M/Q becomes more negative for smaller \bar{r}_0 .

For the general case, the regions in the (c_1, c_2) plane where both bounds are satisfied are plotted for several values of \bar{r}_0 in Figure 6.

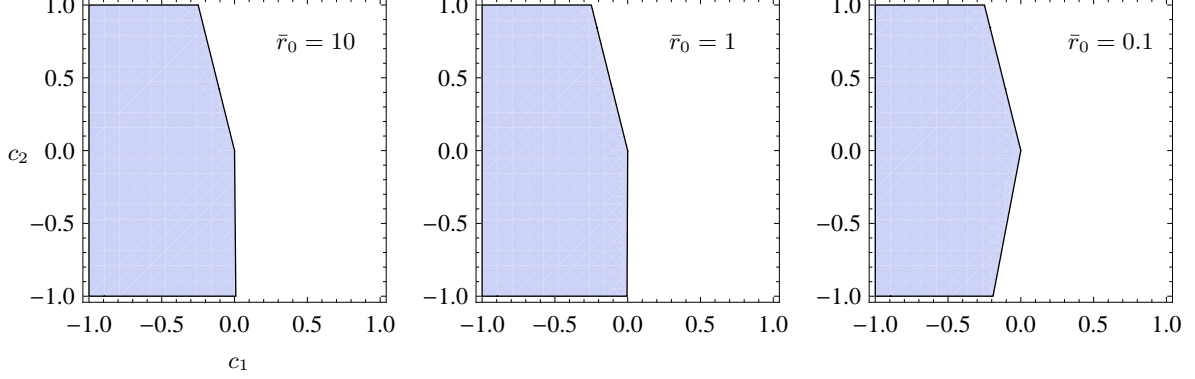


FIG. 6: Here we consider an extremal black hole with a spherical horizon in a six-derivative theory with $b_1 = b_2 = b_3 = 0$. The shaded regions represent points in the (c_1, c_2) plane (for various values of \bar{r}_0) where both the WGC and the KSS bound are satisfied simultaneously.

Appendix D: Expressions for the Effective Action

In this appendix we give the coefficients of the effective action (3.7) obtained from the action (2.6), which are needed for the computation of the viscosity (3.15). Substituting the ansatz given in (2.26) yields

$$A(r) = \frac{2r^3\omega}{\sigma} \left[1 - \frac{2b_1(\sigma(6\omega + 6r\omega' + r^2\omega'') - r\sigma'(6\omega + r\omega'))}{r^2\sigma^3} - \frac{4b_2(\omega + r\omega')}{r^2\sigma^2} + \frac{3c_1(\sigma\omega' - 2\omega\sigma')^2}{r^2\sigma^6} \right. \\ \left. + \frac{b_3(r\sigma'(8\omega + r\omega') - \sigma(4\omega + 5r\omega' + r^2\omega''))}{2r^2\sigma^3} + \frac{3c_2(\sigma\omega'(4\omega + r^2\omega'') - \sigma'(8\omega^2 + r^2\omega'^2))}{4r^3\sigma^5} \right] \quad (D1)$$

$$B(r) = \frac{3r^3\omega}{2\sigma} \left[1 + \frac{2b_1(r\sigma'(6\omega + r\omega') - \sigma(6\omega + 6r\omega' + r^2\omega''))}{r^2\sigma^3} \right. \\ - \frac{2b_2(12\sigma\omega^2 - 6r\omega^2\sigma' + 14r\sigma\omega\omega' - 3r^2\omega\sigma'\omega' + r^2\sigma(\omega'\omega)')}{3r^2\sigma^3\omega} \\ + \frac{b_3(r^2\omega^2\sigma'^2 + r\sigma\omega\sigma'(4\omega - r\omega') + \sigma^2(r^2\omega'^2 - r\omega\omega' + \omega(\omega - r^2\omega'')))}{3r^2\sigma^4\omega} \\ + \frac{2c_1}{r^4\sigma^7\omega} (4r^3\omega^3\sigma'^3 - 6r^2\sigma\omega^2\sigma'^2(2\omega + r\omega') + r\sigma^2\omega\sigma'(16\omega^2 + 12r\omega\omega' + 3r^2\omega'^2) \\ - \sigma^3(2\omega^3 + 8r\omega^2\omega' + 3r^2\omega\omega'^2 + r^3\omega'^3)) \\ + \frac{c_2}{4r^4\sigma^6\omega} (r\sigma\sigma'(2r^2\omega\omega'^2 - 16\omega^3 + r^3\omega'^3 + 2r^3\omega\omega'\omega'') - 2r^4\omega\sigma'^2\omega'^2 \\ \left. + \sigma^2(24\omega^3 - r^4\omega'^2\omega'' - 2r\omega\omega'(r^2\omega'' - 4\omega)) \right) \quad (D2)$$

$$E(r) = \frac{\omega^2}{2\sigma^3} \left[b_3 r^3 - \frac{12 c_1 r^2 (\sigma \omega' - 2\omega \sigma')}{\sigma^3} + \frac{3 c_2 \omega r}{\sigma^2} \right] \quad (D3)$$

$$F(r) = -\frac{2\omega}{\sigma^3} \left[r^2 b_2 (2\omega + r \omega') - \frac{r^2 b_3 (3\sigma \omega - r \omega \sigma' + r \sigma \omega')}{2\sigma} + \frac{3 r c_1 (\sigma \omega' - 2\omega \sigma') (4\sigma \omega - 2 r \omega \sigma' + r \sigma \omega')}{\sigma^4} \right. \\ \left. + \frac{3 c_2 (r^3 \sigma \omega' \omega'' - 8 \sigma \omega^2 - r^3 \sigma' \omega'^2)}{8 \sigma^3} \right] \quad (D4)$$

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- [1] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, “The string landscape, black holes and gravity as the weakest force,” JHEP **0706**, 060 (2007) [arXiv:hep-th/0601001].
- [2] P. Kovtun, D. T. Son and A. O. Starinets, “Viscosity in strongly interacting quantum field theories from black hole physics,” Phys. Rev. Lett. **94**, 111601 (2005) [arXiv:hep-th/0405231].
- [3] C. Vafa, “The string landscape and the swampland,” arXiv:hep-th/0509212.
- [4] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, “Viscosity Bound Violation in Higher Derivative Gravity,” Phys. Rev. D **77**, 126006 (2008), [arXiv:0712.0805 [hep-th]].
- [5] S. Cremonini, J. T. Liu and P. Szepietowski, “Higher Derivative Corrections to R-charged Black Holes: Boundary Counterterms and the Mass-Charge Relation,” JHEP **1003**, 042 (2010) [arXiv:0910.5159 [hep-th]].
- [6] Y. Kats and P. Petrov, “Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory,” JHEP **0901**, 044 (2009) [arXiv:0712.0743 [hep-th]].
- [7] R. C. Myers, M. F. Paulos and A. Sinha, “Holographic Hydrodynamics with a Chemical Potential,” JHEP **0906**, 006 (2009) [arXiv:0903.2834 [hep-th]].
- [8] S. Cremonini, K. Hanaki, J. T. Liu and P. Szepietowski, “Higher derivative effects on eta/s at finite chemical potential,” Phys. Rev. D **80**, 025002 (2009) [arXiv:0903.3244 [hep-th]].
- [9] S. S. Pal, “Weak Gravity Conjecture, Central Charges and η/s ,” arXiv:1003.0745 [hep-th].
- [10] Y. Kats, L. Motl and M. Padi, “Higher-order corrections to mass-charge relation of extremal black holes,” JHEP **0712**, 068 (2007) [arXiv:hep-th/0606100].
- [11] T. Banks, M. Johnson and A. Shomer, “A note on gauge theories coupled to gravity,” JHEP **0609**, 049 (2006) [arXiv:hep-th/0606277].
- [12] A. Giveon and D. Gorboson, “On black fundamental strings,” JHEP **0610**, 038 (2006) [arXiv:hep-th/0606156].
- [13] A. Giveon, D. Gorboson and M. Stern, “Fundamental Strings and Higher Derivative Corrections to d-Dimensional Black Holes,” JHEP **1002**, 012 (2010) [arXiv:0909.5264 [hep-th]].
- [14] A. Buchel, J. T. Liu and A. O. Starinets, “Coupling constant dependence of the shear viscosity in N=4 supersymmetric Yang-Mills theory,” Nucl. Phys. B **707**, 56 (2005) [arXiv:hep-th/0406264].
- [15] A. Buchel, “Resolving disagreement for eta/s in a CFT plasma at finite coupling,” Nucl. Phys. B **803**, 166 (2008) [arXiv:0805.2683 [hep-th]].
- [16] R. C. Myers, M. F. Paulos and A. Sinha, “Quantum corrections to eta/s,” Phys. Rev. D **79**, 041901 (2009) [arXiv:0806.2156 [hep-th]].
- [17] X. H. Ge, Y. Matsuo, F. W. Shu, S. J. Sin and T. Tsukioka, “Viscosity Bound, Causality Violation and Instability with Stringy Correction and Charge,” JHEP **10**, 009 (2008), [arXiv:0808.2354 [hep-th]].
- [18] G. Koutsoumbas, E. Papantonopoulos, G. Siopsis, “Shear Viscosity and Chern-Simons Diffusion Rate from Hyperbolic Horizons,” Phys. Lett. B **677**, 74 (2009), [arXiv:0809.3388 [hep-th]].
- [19] R. G. Cai, Z. Y. Nie and Y. W. Sun, “Shear Viscosity from Effective Couplings of Gravitons,” Phys. Rev. D **78**, 126007 (2008) [arXiv:0811.1665 [hep-th]].
- [20] “A. Buchel, R. C. Myers and A. Sinha, Beyond $\eta/s = 1/4\pi$,” JHEP **03**, 084 (2009), [arXiv:0812.2521 [hep-th]].
- [21] R. G. Cai, Z. Y. Nie, N. Ohta and Y. W. Sun, “Shear Viscosity from Gauss-Bonnet Gravity with a Dilaton Coupling,” Phys. Rev. D **79**, 066004 (2009) [arXiv:0901.1421 [hep-th]].
- [22] A. Ghodsi and M. Alishahiha, “Non-relativistic D3-brane in the presence of higher derivative corrections,” Phys. Rev. D **80**, 026004 (2009) [arXiv:0901.3431 [hep-th]].
- [23] A. Sinha and R. C. Myers, “The viscosity bound in string theory,” Nucl. Phys. A **830**, 295C (2009) [arXiv:0907.4798 [hep-th]].
- [24] S. S. Pal, “ η/s at finite coupling,” Phys. Rev. D **81**, 045005 (2010), [arXiv:0910.0101 [hep-th]].
- [25] J. de Boer, M. Kulaxizi and A. Parnachev, “AdS₇/CFT₆, Gauss-Bonnet Gravity, and Viscosity Bound,” JHEP **1003**, 087 (2010) [arXiv:0910.5347 [hep-th]].
- [26] X. O. Camanho and J. D. Edelstein, “Causality constraints in AdS/CFT from conformal collider physics and Gauss-Bonnet gravity,” JHEP **1004**, 007 (2010) [arXiv:0911.3160 [hep-th]].
- [27] X. O. Camanho and J. D. Edelstein, “Causality in AdS/CFT and Lovelock theory,” JHEP **1006**, 099 (2010) [arXiv:0912.1944 [hep-th]].
- [28] R. Brustein and A. J. M. Medved, “Proof of a universal lower bound on the shear viscosity to entropy density ratio,” [arXiv: 0908.1473 [hep-th]].

- [29] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, “The Viscosity Bound and Causality Violation,” *Phys. Rev. Lett.* **100**, 191601 (2008) [arXiv:0802.3318 [hep-th]].
- [30] N. Banerjee and S. Dutta, “Near-Horizon Analysis of η/s ,” arXiv:0911.0557 [hep-th].
- [31] R. C. Myers and B. Robinson, “Black Holes in Quasi-topological Gravity,” arXiv:1003.5357 [gr-qc].
- [32] R. C. Myers, M. F. Paulos and A. Sinha, “Holographic studies of quasi-topological gravity,” arXiv:1004.2055 [hep-th].
- [33] D. M. Hofman and J. Maldacena, “Conformal collider physics: Energy and charge correlations,” *JHEP* **0805**, 012 (2008) [arXiv:0803.1467 [hep-th]].
- [34] R. R. Metsaev and A. A. Tseytlin, “Curvature Cubed Terms In String Theory Effective Actions,” *Phys. Lett. B* **185**, 52 (1987).
- [35] L. F. Abbott and S. Deser, “Stability of gravity with a cosmological constant,” *Nucl. Phys. B* **195**, 76 (1982).
- [36] S. Deser and B. Tekin, “Gravitational energy in quadratic curvature gravities,” *Phys. Rev. Lett.* **89**, 101101 (2002) [arXiv:hep-th/0205318].
- [37] S. Deser and B. Tekin, “Energy in generic higher curvature gravity theories,” *Phys. Rev. D* **67**, 084009 (2003) [arXiv:hep-th/0212292].
- [38] R. L. Arnowitt, S. Deser, and C. W. Misner, “Dynamical structure and definition of energy in general relativity,” *Phys. Rev.* **116**, 1322 (1959);
“Canonical variables for general relativity,” *Phys. Rev.* **117**, 1595 (1960);
“The dynamics of general relativity,” in *Gravitation: an Introduction to Current Research*, L. Witten ed. (Wiley 1962), pp 227-265.
- [39] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press, Oxford (1975).
- [40] V. Balasubramanian and P. Kraus, “A stress tensor for anti-de Sitter gravity,” *Commun. Math. Phys.* **208**, 413 (1999) [arXiv:hep-th/9902121].
- [41] M. Henningson and K. Skenderis, “The holographic Weyl anomaly,” *JHEP* **9807**, 023 (1998) [arXiv:hep-th/9806087].
- [42] S. W. Hawking, “The path integral approach to quantum gravity,” in *General Relativity: An Einstein Centenary Survey*, eds. S. W. Hawking and W. Israel, Cambridge University Press, Cambridge (1979).
- [43] M. Henneaux and C. Teitelboim, “Asymptotically Anti-De Sitter Spaces,” *Commun. Math. Phys.* **98**, 391 (1985).
- [44] G. Barnich and F. Brandt, “Covariant theory of asymptotic symmetries, conservation laws and central charges,” *Nucl. Phys. B* **633**, 3 (2002) [arXiv:hep-th/0111246].
- [45] G. Barnich and G. Compère, “Surface charge algebra in gauge theories and thermodynamic integrability,” *J. Math. Phys.* **49**, 042901 (2008) [arXiv:0708.2378 [gr-qc]].
- [46] G. Compère, “Symmetries and conservation laws in Lagrangian gauge theories with applications to the mechanics of black holes and to gravity in three dimensions,” arXiv:0708.3153 [hep-th].
- [47] J. Lee and R. M. Wald, “Local symmetries and constraints,” *J. Math. Phys.* **31**, 725 (1990).
- [48] V. Iyer and R. M. Wald, “Some properties of Noether charge and a proposal for dynamical black hole entropy,” *Phys. Rev. D* **50**, 846 (1994) [arXiv:gr-qc/9403028].
- [49] R. M. Wald and A. Zoupas, “A General Definition of ”Conserved Quantities” in General Relativity and Other Theories of Gravity,” *Phys. Rev. D* **61**, 084027 (2000) [arXiv:gr-qc/9911095].
- [50] D. T. Son and A. O. Starinets, “Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications,” *JHEP* **0209**, 042 (2002) [arXiv:hep-th/0205051].
- [51] G. Policastro, D. T. Son and A. O. Starinets, “From AdS/CFT correspondence to hydrodynamics,” *JHEP* **0209**, 043 (2002) [arXiv:hep-th/0205052].
- [52] N. Banerjee and S. Dutta, “Higher Derivative Corrections to Shear Viscosity from Graviton’s Effective Coupling,” *JHEP* **0903**, 116 (2009) [arXiv:0901.3848 [hep-th]].
- [53] N. Iqbal and H. Liu, “Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm,” *Phys. Rev. D* **79**, 025023 (2009), [arXiv:0809.3808 [hep-th]].
- [54] R. M. Wald, “Black hole entropy is the Noether charge,” *Phys. Rev. D* **48**, 3427 (1993) [arXiv:gr-qc/9307038].
- [55] T. Jacobson, G. Kang and R. C. Myers, “On Black Hole Entropy,” *Phys. Rev. D* **49**, 6587 (1994) [arXiv:gr-qc/9312023].
- [56] R. R. Metsaev and A. A. Tseytlin, “Order alpha-prime (Two Loop) Equivalence of the String Equations of Motion and the Sigma Model Weyl Invariance Conditions: Dependence on the Dilaton and the Antisymmetric Tensor,” *Nucl. Phys. B* **293**, 385 (1987).
- [57] D. M. Hofman, “Higher Derivative Gravity, Causality and Positivity of Energy in a UV complete QFT,” *Nucl. Phys. B* **823**, 174 (2009) [arXiv:0907.1625 [hep-th]].